Distributed Geometric-Programming-Based Power Control in Cellular Cognitive Radio Networks

Qingqing Jin, Dongfeng Yuan, Zhangyu Guan
Wireless Mobile Communication and Transmission (WMCT) Lab
School of Information Science and Engineering, Shandong University
Jinan, Shandong, China, 250100
{qingqingjin, dfyuan, zhangyuguan}@sdu.edu.cn

***Abstract*—Power control is critical for wireless communications that allow spectrum sharing among secondary users and primary users. In this paper, we derive an optimal distributed power control strategy aiming at the total capacity maximization of secondary network with interference constraints to primary users. Due to the nonconvexity of system utility, geometric programming is introduced to transform nonconvex optimization problems into convex optimization problems. Furthermore, system utility is usually coupled which means each utility depends not only on its local variables but also on the variables of other utilities. We introduce auxiliary variables and extra equality constraints to transfer the coupling in utility to coupling in constraints. The solution of the proposed power control strategy is shown to be globally optimal and leads to excellent performance.**

I. Introduction

With the rapid emergence of wireless terminals and services, more and more spectrum is demanded and the radio resources become increasingly scarce. However, according to a report published by the Federal Communication Commission (FCC), the radio spectrum is occupied spatially and temporarily with an inefficient utilization[1]. Cognitive Radio (CR) [2], [3] has been considered as the key technology to solve this problem since it enables unlicensed users to operate in licensed bands without permission, which is referred to as *spectrum sharing*.

The realization of the spectrum sharing can be classified as two types. One is based on spectrum sensing, detecting spectrum holes and access the idle bands adaptively. The other is based on interference temperature model which allows the coexisttence of Primary Users (PUs) and Secondary Users (SUs) in the same band. Related work on the latter mode has appeared in [4] and [5]. In [4], both quality of service (QoS) of SUs and the interference temperature constraints at a particular Primary Receiver (PR) are considered in spectrum sharing among a group of spread spectrum users. In [5], a payoff function including tax factor and penalty factor is formulated to maximize the throughput of the secondary network with multiple PRs, but the solution can not guarantee global optimal.

In CR networks, an important topic of research is to maximize the total utilities of SUs without penalizing the PUs, and it is common practice to formulate this maximization as convex optimization. However, sometimes the utility functions are not necessarily concave or convex with respect to the transmit power. So the power control optimization is probably nonlinear and belongs to NP-hard problems. To solve this problems, Geometric programming (GP) has been introduced into communication systems as it can transform these nonconvex power control problems into convex optimizations through making logarithmic operation to variables. Then the global optimal solution can be efficiently obtained [6], [7]. However, conventional GP usually requires collection of global system information and centralized computation. This might result in drastic increase in communication overhead and impose high burden of computation to the base station. As thus, the GP cannot be directly applied in some scenarios where users work in a distributed way, such as the CR networks investigated in this paper.

To alleviate above problem, we use dual decomposition method to decompose a GP problem into several subproblems such that the original resource allocation can be solved respectively. In CR networks, it is shown later in this paper that the utility with respect to the transmit power is indeed coupled, or in other words, each user's utility depends not only on its own power but also on the power of other users. To remove these coupling, we introduce auxiliary variables and extra equality constraints such that these coupling in utility are transferred to coupling in constraints. Then the utility can be decoupled through dual decomposition[8], [9].

The rest of this paper is organized as follows. Section II describes the system model of cognitive radio network. In section III, we give a brief introduction of GP method and coupled utility decomposition. In section IV, the system utility optimization problem is formulated and the optimal distributed power control algorithm is proposed. Finally, the performance of the proposed algorithm is investigated by numerical results in section V.

II. System Model of Cognitive Radio Network

As an illustration of CR networks, we study the CDMA cellular network as the secondary network with a single base station (BS) and N unlicensed users. There are M PRs located in fixed positions outside of the secondary network. Recall that in CR networks, PUs and SUs operate in the

This work was supported by NSFC (No. 60672036, No. 60832008) and Key Project of Provincial Scientific Foundation of Shandong (No. Z2008G01).

same spectrum band. Therefore the channels are inherently interference channels. Denote h_{ij} as the channel gain from transmitter j to receiver i. Since all the receivers are mounted in the same BS, the first subscript i can be omitted for denotation convenience. Then h_j means the channel gain from transmitter j to any receiver in the BS. Denote γ_i as the uplink signal-to-interference-and-noise ratio (SINR) of user i, and is represented as [10]:

$$\gamma_i(\mathbf{p}) = \frac{W}{R} \cdot \frac{h_i p_i}{\sum_{j=1, j\neq i}^{N} h_j p_j + \sigma^2}, \qquad \forall i \tag{1}$$

where W and R are spread spectrum bandwidth and transmit rate respectively. $\mathbf{p} = (p_1, p_2, \cdots, p_N)$ is the transmit power vector in which $p_i, i \in \{1, 2, \cdots, N\}$ are transmit powers of user i. σ^2 is the background noise which is assumed to be AWGN.

Denote I_m as the maximum interference tolerance at PR m, which characterizes the "worst case" of the RF environment. Then I_m can be given as

$$I_m = \xi T_m \tag{2}$$

where ξ is the Boltzman's constraint, T_m is the interference temperature limit. So the main constraint of power control in CR networks is to make the interference caused by SUs to the PUs below I_m, i.e.,

$$\sum_{i=1}^{N} g_{mi} p_i \leq I_m, \qquad \forall m \tag{3}$$

where g_{mi} is the channel gain from SU i to PR m.

III. Geometric Programming and Coupled Utility Decomposition

In this section, we first briefly review some basic definitions of GP, and then show how to decompose the coupled utilities using dual decomposition method.

A. Geometric Programming

GP are usually formulated in two equivalent optimization forms. One is *standard GP* with posynomial objective function, and the other is *convex GP* which is the transformation of standard GP. Next, before defining the two forms, we first introduce the concepts of monomial and posynomial.

Definition 1 (monomial): A function $f : \mathbf{R}^n \rightarrow \mathbf{R}$ is a monomial if it has the following expression:

$$g(\mathbf{x}) \quad = c x_1^{a^{(1)}} x_2^{a^{(2)}} \ldots x_n^{a^{(n)}} \tag{4}$$

where the multiplicative constant c is nonnegative and the exponential constants $a^{(j)} \in \mathbf{R}, j = 1, 2, \ldots n$.

Definition 2 (posynomial): A function $f : \mathbf{R}^n \rightarrow \mathbf{R}$ is a posynomial if it is a sum of monomial, as the following below:

$$f(\mathbf{x}) \quad = \sum_{m=1}^{M} c_m x_1^{a_m^{(1)}} x_2^{a_m^{(2)}} \ldots x_n^{a_m^{(n)}} \tag{5}$$

where the $c_m, m = 1, 2, \ldots M$ are nonnegative and $a_m^{(j)} \in \mathbf{R}, j = 1, 2, \ldots n, m = 1, 2, \ldots M$.

Standard GP is the optimization with posynomial objective function and constraints including posynomial inequalities with upper bound and monomial equalities.

$$\begin{aligned} \text{minimize} \quad & f_0(\mathbf{x}) \\ \text{subject to} \quad & f_i(\mathbf{x}) \leq 1, \qquad \forall i \\ & g_l(\mathbf{x}) = 1, \qquad \forall l \end{aligned} \tag{6}$$

where $g_l(\mathbf{x}), f_i(\mathbf{x})$ are monomial and posynomial defined in (4) and (5), respectively.

Notice that due to the nonconvex property of posynomials, standard GP is not a convex optimization problem. However, through a logarithmic operation to variables, constants and functions, we can transform standard GP into convex GP which is a convex optimization problem.

$$\begin{aligned} \text{minimize} \quad & r_0(\mathbf{y}) = \log \Sigma_{m=1}^{M_0} \exp(\mathbf{a}_{0m}^T \mathbf{y} + b_{0m}) \\ \text{subject to} \quad & r_i(\mathbf{y}) = \log \Sigma_{m=1}^{M_i} \exp(\mathbf{a}_{im}^T \mathbf{y} + b_{im}) \leq 0, \forall i \\ & s_l(\mathbf{y}) = \mathbf{a}_l^T \mathbf{y} + b_l = 0, \qquad \forall l \end{aligned} \tag{7}$$

where $b_{im} = \log c_{im}, b_l = \log c_l$, $\mathbf{y} = (y_1, y_2, \cdots, y_n)$ in which $y_i = \log x_i$, $\mathbf{a}_{im} = (a_m^{(1)}, a_m^{(2)}, \cdots, a_m^{(n)})$ and the convex property of the problem can be readily testified because the functions are in log-sum-exp forms[11].

B. Decomposition of coupled utility

In sum utility maximization (SUM), it is usually assumed that the utilities are uncoupled, such that the maximization can be solved by conventional method such as dual decomposition. However, this assumption cannot always be satisfied in cooperative or competitive resource allocations. Following maximization shows an example of coupled utilities.

$$\begin{aligned} \text{maximize} \quad & \sum_{i=1}^{N} U_i(\mathbf{x}_i, \{\mathbf{x}_j\}_{j \in \mathcal{S}(i)}) \\ \text{subject to} \quad & \mathbf{x}_i \in \chi_i \qquad \forall i \end{aligned} \tag{8}$$

where the utilities U_i which are strictly concave relate to local variable $\mathbf{x}_i$ and other utility variable $\mathbf{x}_j$ for $j \in \mathcal{S}(i)$. $\mathcal{S}(i)$ is the set of indexes of utilities coupled with the ith utility.

To solve above problem, it is common practice to introduce auxiliary variables and extra equality constraints to transfer the coupling in utility to coupling in constraints. Consequently, the coupling can be decoupled by dual decomposition and solved through extra consistency prices. In particular, in competitive resource allocation the utilities coupled with each other through an interference term, which consists of the functions of coupling variables. So we introduce auxiliary variables $\mathbf{z}_i$ for the coupled variables and extra inequality constraints to enforce consistency. Then (8) rewrites

$$\begin{aligned} \text{maximize} \quad & \sum_{i=1}^{N} U_i(\mathbf{x}_i, \mathbf{z}_i) \\ \text{subject to} \quad & \mathbf{x}_i \in \chi_i \qquad \forall i \\ & \mathbf{z}_i \geq \sum_{j \in \mathcal{S}(i)} \mathbf{h}_{ij}(x_j) \qquad \forall i \end{aligned} \tag{9}$$

where $\mathbf{x}_i$ and $\mathbf{z}_i$ are local variables of ith utility and the $\mathbf{h}_{ij}$'s are convex functions. The interference inequality constraints can also be substituted by equality constraints if the functions $\mathbf{h}_{ij}$ are linear. In order to employ a distributed algorithm, we take a dual decomposition:

$$\begin{aligned} \text{maximize} \quad & \sum_{i=1}^{N} U_i(\mathbf{x}_i, \mathbf{z}_i) \\ & + \sum_{i=1}^{N} \zeta_i^T \Bigg(\mathbf{z}_i - \sum_{j \in \mathcal{S}(i)} \mathbf{h}_{ij}(x_j) \Bigg) \\ \text{subject to} \quad & \mathbf{x}_i \in \chi_i \quad \forall i \end{aligned} \tag{10}$$

where ζ_i is the Lagrange multiplier, also called consistency price for extra equality constraint. We observe that (10) can be decomposed into several subproblems, which can be solved independently and locally.

Defining $g(\{\zeta_i\})$ the dual function obtained as the optimal value of the Lagrangian solved in (10) for a given set of $\{\zeta_i\}$, and the dual problem is thus given by

$$\text{minimize} \quad g(\{\zeta_i\}) \tag{11}$$

which can be solved with the following updates:

$$\zeta_i(t+1) = \zeta_i(t) - \eta \Bigg(\mathbf{z}_i - \sum_{j \in \mathcal{S}(i)} \mathbf{h}_{ij}(x_j) \Bigg), j \in \mathcal{S}(i) \tag{12}$$

where $\eta > 0$ is a step size and t is the iterative time.

IV. Distributed Geometric-Programming-Based Power Control

One challenge research in cognitive radio networks is how to maximize the utilities of SUs without extra interference to PUs. In this paper, we focus on maximizing total capacity of secondary network with interference constraints to PUs. So the utility of secondary user i is formulated as $u_i(\mathbf{p}) = \log \gamma_i(\mathbf{p})$ in high SINR regime, which is guaranteed in CDMA cellular network with a large spreading gain. Therefore, the objective function of system is $\sum u_i(\mathbf{p})$. Then, the system utility maximization problem can be formulated as:

$$\begin{aligned} \text{maximize} \quad & \sum_{i=1}^{N} u_i(\mathbf{p}) \\ \text{subject to} \quad & \sum_{i=1}^{N} g_{mi} p_i \leq I_m, \quad \forall m \\ & 0 \leq p_i \leq p_{max}, \quad \forall i \end{aligned} \tag{13}$$

where the optimization variable is the transmit power vector $\mathbf{p} = (p_1, p_2, \ldots p_N)$. The first constraint indicates that the aggregate interference from SUs to PR m is restricted by the interference constraint I_m. The other constraint prescribes the minimum and maximum value of transmit power of each user.

The problem (13) is a nonlinear, nonconvex optimization problem whose solution can not be guaranteed global optimal. So we introduce GP method to solve it. First, maximizing $\sum \log(\gamma_i(\mathbf{p}))$ is equivalent to maximizing $\log \prod \gamma_i(\mathbf{p})$, which is then equivalent to maximizing $\prod \gamma_i(\mathbf{p})$. As the $\prod \gamma_i(\mathbf{p})$ is not a posynomial, we reverse the objective function and obtain the standard GP problem:

$$\begin{aligned} \text{minimize} \quad & \prod_{i=1}^{N} \frac{1}{\gamma_i(\mathbf{p})} \\ \text{subject to} \quad & \frac{1}{I_m} \sum_{i=1}^{N} g_{mi} p_i \leq 1, \quad \forall m \\ & \frac{p_i}{p_{max}} \leq 1, \quad \forall i \end{aligned} \tag{14}$$

Next, the standard GP problem can be turned into convex GP problem through a logarithmic operation to the variables $\hat{p}_i = \log p_i$ and a logarithmic operation to the function. The transformation of objective function is shown as following:

$$\begin{aligned} \textstyle\prod_{i=1}^{N} \frac{1}{\gamma_i(\mathbf{p})} &= \prod_{i=1}^{N} \frac{\sum_{j \neq i} h_j p_j + \sigma^2}{G h_i p_i} \\ &= \sum_{i=1}^{N} \log \Bigg(\frac{\exp(-\hat{p}_i)}{G h_i} \big(\sum_{j \neq i} h_j \exp(\hat{p}_j) + \sigma^2 \big) \Bigg) \end{aligned} \tag{15}$$

where $G = W/R$ denote the spreading gain.

The transformed objective function which is strictly convex can be considered as a new expression of aggregate "user utilities", each of which depends not only on local power $\hat{p}_i$ but also on other powers $\{\hat{p}_j\}_{j \neq i}$. In order to decouple the coupling in "user utilities", we need to introduce special auxiliary variables and equality constraints. In a distributed strategy, the minimization of the objective function (15) with constraints would require each user have knowledge of interfering channels and interfering transmit powers, which would result in a large amount of message exchange. For the efficiency of strategy, we assume that every user i has the capability to estimate the interference $Z_i = \sum_{j \neq i} h_j p_j$ from other users and keep a copy locally. Then We choose $\{Z_i\}$ as auxiliary variables and the problem (13) can be reformulated in convex GP form as following below:

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{N} \log \Bigg(\frac{\exp(-\hat{p}_i)}{G h_i} \Big(\exp(\hat{Z}_i) + \sigma^2 \Big) \Bigg) \\ \text{subject to} \quad & \sum_{i=1}^{N} g_{mi} \exp(\hat{p}_i) - I_m \leq 0, \quad \forall m \\ & \hat{p}_i - \log p_{max} \leq 0, \quad \forall i \\ & \sum_{j \neq i} h_j \exp(\hat{p}_j) - \exp(\hat{Z}_i) = 0 \quad \forall i \end{aligned} \tag{16}$$

We relax the constraints to the objective function, and the

Lagrange function is:

$$\begin{aligned} & L\Big(\{\hat{p}_i\},\{\hat{Z}_i\},\mu,\nu,\{\zeta_i\}\Big) \\ = & \sum_{i=1}^{N}\log\left(\frac{\exp(-\hat{p}_i)}{Gh_i}\Big(\exp(\hat{Z}_i)+\sigma^2\Big)\right) \\ + & \sum_{i=1}^{N}\mu_i\big(\hat{p}_i-\log p_{max}\big) \\ + & \sum_{m=1}^{M}\nu_m\Big(\sum_{i=1}^{N}g_{mi}\exp(\hat{p}_i)-I_m\Big) \\ + & \sum_{i=1}^{N}\zeta_i\Big(\sum_{j\neq i}h_j\exp(\hat{p}_j)-\exp(\hat{Z}_i)\Big) \end{aligned} \quad (17)$$

where μ and ν are Lagrange multipliers and $\{\zeta_i\}$ are the consistency prices. Due to the decomposability of Lagrange function, it can separate into N subproblems, each of which as following below:

$$\begin{aligned} & L_i\Big(\hat{p}_i,\hat{Z}_i,\mu_i,\nu,\zeta_i\Big) \\ = & \log\left(\frac{\exp(-\hat{p}_i)}{Gh_i}\Big(\exp(\hat{Z}_i)+\sigma^2\Big)\right) \\ + & \mu_i\hat{p}_i+\sum_{m=1}^{M}\nu_m g_{mi}\exp(\hat{p}_i) \\ + & \Big(\sum_{j\neq i}\zeta_j h_i\Big)\exp(\hat{p}_i)-\zeta_i\exp(\hat{Z}_i) \end{aligned} \quad (18)$$

where the consistency prices $\{\zeta_j\}_{j\neq i}$ are obtained from other users through the BS. Then, (18) can be solve by each user separately. And the dual problem is:

$$\begin{aligned} \text{maximize} \quad & D\big(\mu,\nu,\{\zeta_i\}\big) \\ \text{subject to} \quad & \mu\geq 0 \qquad \nu\geq 0 \end{aligned} \quad (19)$$

where $D\big(\mu,\nu,\{\zeta_i\}\big)=\text{minimize } L\big(\{\hat{p}_i\},\{\hat{Z}_i\},\mu,\nu,\{\zeta_i\}\big)$ is the dual function.

The dual problem can be solved using subgradient method which updates the Lagrange multipliers and consistency prices as following:

$$\mu_i(t) = \Big[\mu_i(t-1)+\alpha(t)\big(\hat{p}_i(t)-\log p_{max}\big)\Big]^+ \quad (20)$$

$$\nu_m(t) = \Big[\nu_m(t-1)+\beta(t)\big(\sum_{i=1}^{N}g_{mi}\exp(\hat{p}_i(t))-I_m\big)\Big]^+ \quad (21)$$

$$\zeta_i(t) = \zeta_i(t-1)+\eta(t)\Big(\sum_{j\neq i}h_j p_j(t)-\exp(\hat{Z}_i)\Big) \quad (22)$$

where $\alpha(t)$, $\beta(t)$ and $\eta(t)$ are step sizes which must be positive. $[X]^+=\max\{X,0\}$ and t is the iterative time. The $Z_i(t)=\sum_{j\neq i}h_j p_j(t)$ is the estimated value of total interference from other users to user i.

Based on the KKT necessary condition, the optimal transmit power $\{\hat{p}_i^*\}$ can be calculated by each user separately through the following equation:

$$\frac{\partial L_i\big(\hat{p}_i,\hat{Z}_i,\mu_i,\nu,\zeta_i\big)}{\partial\hat{p}_i}=0, \qquad \forall i \quad (23)$$

and the solution is

$$p_i^*=\exp(\hat{p}_i^*)=\left[\frac{1-\mu_i}{\sum_{m=1}^{M}\nu_m g_{mi}+\sum_{j\neq i}\zeta_j h_i}\right]^+ \quad \forall i \quad (24)$$

The μ_i and ζ_i are updated simultaneously at transmitter i through iterative function (20) and (22), and the value of ζ_i is transmitted to BS with the power solution p_i. The $\{\nu_m\}$ are updated at BS through iterative function (21) with the information of $\{p_i\}$, and then the value of $\{\nu_m\}$ and $\{\zeta_i\}$ are broadcasted to all the users by BS.

We summarize the Distributed Geometric-Programming-Based Power Control (DGPPC) Algorithm as follow:

Initialization:
t=0;
$0\leq p_i(0)\leq p_{max}$, $\mu_i(0)>0$, $\zeta_i(0)>0$; $\forall i$
$\nu_m(0)>0$; $\forall m$

Algorithm at user i: at each iteration t, t=1,2,...

1) receive $\{\nu_m(t-1)\}$ and $\{\zeta_j(t-1)\}_{j\neq i}$ from BS;
2) obtain the power solution based on (24)

$$p_i(t)=\left[\frac{1-\mu_i(t-1)}{\sum_{m=1}^{M}\nu_m(t-1)g_{mi}+\sum_{j\neq i}\zeta_j(t-1)h_i}\right]^+$$

3) update the μ_i and ζ_i simultaneously for the next iteration

$$\begin{aligned} \mu_i(t) &= \big[\mu_i(t-1)+\alpha(t)\big(\log p_i(t)-\log p_{max}\big)\big]^+ \\ \zeta_i(t) &= \zeta_i(t-1)+\eta(t)\big(\sum_{j\neq i}h_j p_j(t)-\exp(\hat{Z}_i)\big) \end{aligned}$$

4) transmit $\zeta_i(t)$ to BS with $p_i(t)$.

Algorithm at BS: at each iteration t, t=1,2,...

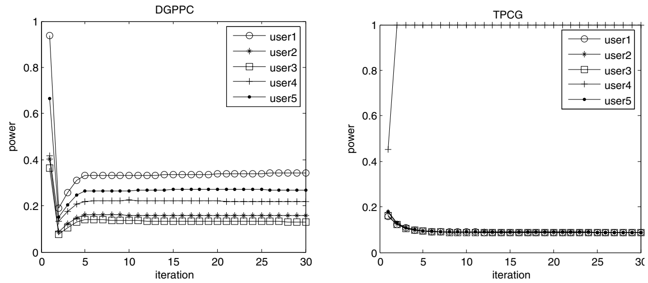
1) update $\{\nu_m\}$ with received power $p_i(t)$ for the next iteration

$$\nu_m(t)=\big[\nu_m(t-1)+\beta(t)\big(\sum_{i=1}^{N}g_{mi}\exp(\hat{p}_i(t))-I_m\big)\big]^+$$

2) broadcast $\{\nu_m(t)\}$ and $\{\zeta_i(t)\}$ to each user.

V. Simulation Results

In this section, we present illustrative numerical results for the proposed power control algorithm. We consider a cellular cognitive radio network with a BS in the center, 5 SUs distributed randomly inside and 3 primary receivers mounted outside. The network covers 500m×500m, and the positions of primary receivers are about 1250m away from the BS. The system parameters are defined as follow: $p_{max}=1$W, $\sigma^2=10^{-10}$W, $W=10^6$Hz and $R=10^4$bit/s. The interference constraints to PUs are set to be the same value, $I_m=5\times10^{-10}$W. The channel gains are figured out using a



(a) (b)

Fig. 1. The power convergence of each user in DGPPC and TPCG.

simple path loss model, $h_i = Kd_i^{-4}$ and $g_{mi} = Kd_{mi}^{-3}$ where K is a constant.

We compare different performances of DGPPC algorithm proposed in this paper with TPCG algorithm proposed in [5]. Figure 1 illustrates the power convergence properties of each user in this two algorithms separately. We can see that, in DGPPC algorithm, each user can converge to its optimal power solution very fast and each solution is low but different from others based on the distance from BS. Therefore, in TPCG algorithm, the nearest user from BS transmits with p_{max} and other users merely obtain very low powers.

In cognitive radio network, two metrics are very significant to be considered: the total capacity of secondary network and the total interference to PUs. Through Figure 2 and Figure 3, it is obviously to conclude that power solutions obtained by DGPPC algorithm cause less interference to PUs but can make secondary network achieve much higher capacity compared with TPCG algorithm solutions.

VI. Conclusion

In this paper, we have studied the power control problem in Cognitive Radio (CR) networks, in which SUs and PUs operate in the same spectrum band. The objective of power control is to maximize the total capacity of SUs subject to the interference constraints to PRs. To obtain the global optimal solution, we introduce the geometric programming method to transform the nonconvex problem into convex optimization problem. Moreover, a special decomposition approach is used to transfer coupling in utility to coupling in constraints. Numerical results indicate that the proposed algorithm converges fast to the global optimal solution. Notice that, in this paper we focus only on the uplink case, the downlink case is not included due to space limitation. In our future research, we wish to take the fairness among SUs into consideration in power control problem.

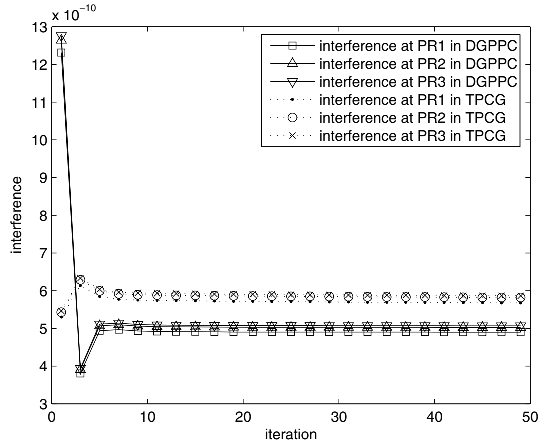


Fig. 2. The interference to each primary receiver in DGPPC and TPCG.

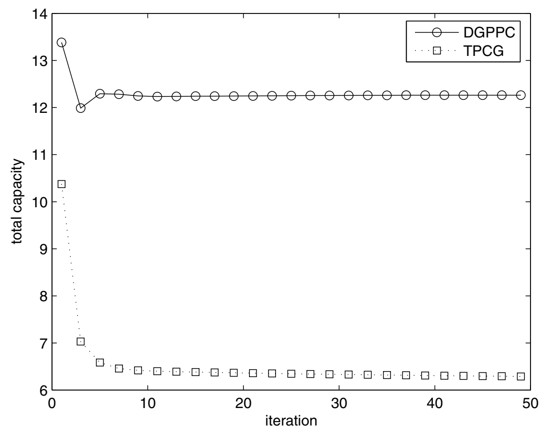


Fig. 3. The total capacity convergence in DGPPC and TPCG.

References

[1] Federal Communication Commission, ”Spectrum Policy Task Force,” Rep. ET Docket no. 02-135, Nov. 2002

[2] J. Mitola, G. Maguire, “Cognitive radio: Making software radios more personal”, *IEEE Personal Communications*, vol. 6, no. 4, pp. 13-18, Aug. 1999

[3] S. Haykin, “Cognitive radio: Brain-empowered wireless communications”, *IEEE Journal on Selected Areas in Communications*, vol. 23, no.2, pp. 201-220, Feb. 2005

[4] Yiping Xing, C. N. Mathur, M. A. Haleem, ”Dynamic Spectrum Access with QoS and Interference Temperature Constraints”, *IEEE Transactions on Mobile Computing*, Volume 6, Issue 4, pp. 423 - 433 Apr. 2007

[5] Hongyan Li, Yibing Gai, Zhiqiang He, Kai Niu, Weiling Wu, ”Optimal Power Control Game Algorithm for Cognitive Radio Networks with Multiple Interference Temperature Limits”, *IEEE Vehicular Technology Conference, 2008*. pp. 1554 - 1558, May 2008

[6] M. Chiang, C. W. Tan, D. P. Palomar, D. O’Neill, D.Julian, ”Power Control By Geometric Programming”, *IEEE Transaction on Wireless Communication*, Volume 6, Issue 7, pp. 2640 - 2651, Jul. 2007

[7] M. Chiang, ”Geometric programming for communication systems”, Foundations and Trends of Communications and Information Theory, vol. 2, no. 1-2, pp. 1-156, Aug. 2005

[8] D. P. Palomar, M. Chiang, ”Alternative Distributed Algorithms for Network Utility Maximization: Framework and Applications”, *IEEE Transactions on Automatic Control*, Volume 52, Issue 12, pp. 2254 - 2269, Dec. 2007

[9] D. P. Palomar, M. Chiang, ”A Tutorial on Decomposition Methods for Network Utility Maximization”, *IEEE Journal on Selected Areas in Communications*, Volume 24, Issue 8, pp. 1439 - 1451, Aug. 2006

[10] C. Saraydar, N. Mandayam, D. Goodman, “Efficient power control via pricing in wireless data networks”, *IEEE Transactions on Communications*, Volume 50, Issue 2, pp. 291-303, Feb. 2002

[11] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004