Distributed Resource Management for Cognitive Ad Hoc Networks With Cooperative Relays

Zhangyu Guan, *Member, IEEE*, Tommaso Melodia, *Member, IEEE*, Dongfeng Yuan, *Senior Member, IEEE*, and Dimitris A. Pados, *Senior Member, IEEE*

***Abstract*—It is well known that the data transport capacity of a wireless network can be increased by leveraging the spatial and frequency diversity of the wireless transmission medium. This has motivated the recent surge of research in *cooperative* and *dynamic-spectrum-access* (which we also refer to as cognitive spectrum access) networks. Still, as of today, a key open research challenge is to design distributed control strategies to dynamically jointly assign: 1) portions of the spectrum and 2) cooperative relays to different traffic sessions to maximize the resulting network-wide data rate. In this paper, we make a significant contribution in this direction. First, we mathematically formulate the problem of joint spectrum management and relay selection for a set of sessions concurrently utilizing an interference-limited infrastructure-less wireless network. We then study distributed solutions to this (nonlinear and nonconvex) problem. The overall problem is separated into two subproblems: 1) spectrum management through power allocation with given relay selection strategy; and 2) relay selection for a given spectral profile. Distributed solutions for each of the two subproblems are proposed, which are then analyzed based on notions from variational inequality (VI) theory. The distributed algorithms can be proven to converge, under certain conditions, to VI solutions, which are also Nash equilibrium (NE) solutions of the equivalent NE problems. A distributed algorithm based on iterative solution of the two subproblems is then designed. Performance and price of anarchy of the distributed algorithm are then studied by comparing it to the globally optimal solution obtained with a newly designed centralized algorithm. Simulation results show that the proposed distributed algorithm achieves performance that is within a few percentage points of the optimal solution.**

***Index Terms*—Cognitive radio networks, cooperative relay, distributed spectrum management, game theory.**

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Z. Guan is with the Department of Electrical and Computer Engineering, Northeastern University, Boston, MA 02115 USA, and also with the Department of Electrical Engineering, State University of New York (SUNY) at Buffalo, Buffalo, NY 14260 USA (e-mail: zguan@ece.neu.edu).

T. Melodia is with the Department of Electrical and Computer Engineering, Northeastern University, Boston, MA 02115 USA (e-mail: melodia@ece.neu.edu).

D. Yuan is with the School of Information Science and Engineering, Shandong University, Jinan 250100, China (e-mail: dfyuan@sdu.edu.cn).

D. A. Pados is with the Department of Electrical Engineering, State University of New York (SUNY) at Buffalo, Buffalo, NY 14260 USA (e-mail: pados@buffalo.edu).

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I. INTRODUCTION

THE CONCEPT of *cooperative communications* has been proposed to achieve spatial diversity without requiring multiple transceiver antennas on a wireless device [2]–[4]. In cooperative communications, in their *virtual multiple-input single-output* (VMISO) variant, each node is equipped with a single antenna and relies on the antennas of neighboring devices to achieve spatial diversity. Thanks to the broadcast nature of the wireless channel, signals transmitted by a source can be overheard by neighboring devices. Therefore, one (or multiple) relays can forward their received signals to the destination. Multiple copies of the original signal can then be received at the destination, which can combine them to decode the original message.

A vast and growing literature on information and communication theoretic results [5], [6] in cooperative communications is available. Readers are referred to [7], [8], and references therein for excellent surveys in this area. Problems addressed include the definition of algorithms to establish when and how to cooperate and optimal cooperative transmission strategies. For example, [3], [9], and [10] study in depth outage probability and capacity of an isolated communication link. However, only simple network topologies are studied, e.g., single source–destination pairs with single and fixed relay nodes, while network-wide interactions among multiple concurrent cooperative communication sessions are not considered. Distributed relay selection algorithms are also proposed based on noncooperative game theory, e.g., auction theory [11] or Stackelberg games [12], [13]. However, typically a single channel and no interference among multiple concurrent communication links is assumed.

In this paper, we focus on cognitive ad hoc networks and look at the fundamental problem of designing algorithms to leverage the spatial and frequency diversity of the wireless channel by jointly allocating portions of the spectrum and cooperative relays to the sessions to maximize the overall achievable data rate. Different from traditional cognitive radio (CR) communications where there are primary and secondary users [14], in this paper we use cognitive to refer to dynamic spectrum access. Through our developments, we make the following contributions.

- *Cooperative networks with dynamic spectrum access*: We study the problem of joint spectrum management and relay assignment in dynamic-spectrum-access cooperative networks with decentralized control.

- *Effect of cooperation and dynamic spectrum access in interference-limited networks*: Since cooperative and cognitive ad hoc networks are inherently interference-limited, and ideal orthogonal FDMA or TDMA channels cannot be easily established without centralized control, we consider a general interference model. The results obtained can be applied to interference-free networks as a special case.
- *Distributed algorithms*: We design and analyze distributed solution algorithms for spectrum assignment and relay selection, based on best-response local optimizations. Since the original joint optimization problem has a rather complex (combinatorial, nonlinear, and nonconvex) mathematical formulation, we decompose it into two separate subproblems, i.e., spectrum access and relay selection. For each subproblem, relying on notions from *variational inequality* (VI) theory [15], we analyze the existence of a Nash equlibrium (NE) of the problem, design distributed solution algorithms, and prove convergence of the algorithm to an NE point.
- *Centralized algorithm*: We propose and study a *centralized* iterative solution algorithm based on a combination of branch-and-bound and convex approximations of nonlinear, nonconvex problem constraints. We show that the algorithm provably converges to the optimal solution of the global problem. We evaluate and compare the distributed solution to the optimal solution.

The proposed algorithm can be directly applied to a scenario with multiple coexisting preestablished source–destination pairs. In addition, it can provide an upper bound to the performance of simpler centralized/distributed algorithms for spectrum management and relay assignment. Lastly, in a multihop ad hoc network, the proposed algorithm can be used to optimally control resource allocation for an independent set of transmissions with primary interference constraints (i.e., no transmitters and receivers in common) periodically scheduled by a separate scheduling algorithm, where idle nodes can be used as potential relays.

The rest of the paper is organized as follows. In Section II, we discuss related work, and in Section III, we introduce system model and problem formulation. Then, we describe and analyze the distributed algorithm in Section IV, and describe the centralized algorithm in Section V. Finally, we present performance evaluation results in Section VI and conclude the paper in Section VII.

II. Related Work

Relay selection in cooperative wireless networks has been an important topic of research [11], [12], [16]–[23]. Shi *et al.* [16] studied the relay selection problem in ad hoc networks and proposed an algorithm with attractive properties of both optimality guarantee and polynomial time complexity. Hou *et al.* [17] investigated the problem of joint flow routing and relay selection in multihop ad hoc networks and proposed an optimal centralized algorithm with arbitrary predefined optimality precision based on the powerful branch-and-cut framework. Rossi *et al.* [18] studied the optimal cooperator selection in ad hoc networks based on Markov decision processes and the focused real time dynamic programming technique. In [19], Yang *et al.* proposed a HERA scheme for cooperative networks to avoid system performance degradation due to the selfish relay selections. In [20], Zhou *et al.* proposed an interference-aware relay selection scheme for two-hop relay networks with multiple source–destination (S–D) pairs. Song *et al.* derived the achievable symbol error rate (SER) and frame error rate (FER) for bidirectional relay networks [24], [25] and for cooperative networks with hybrid forwarding strategies [26], respectively. Different from these works, we jointly study relay selection and spectrum management and focus on distributed algorithms.

Distributed relay selection based on game theory has also attracted significant attention [27]. For example, [11] and [28] studied the problem of distributed relay selection and relay power allocation in cooperative networks based on auction theory. In [12], Wang *et al.* formulated the problem of distributed relay selection in a multiple-relay single-session network as a Stackelberg game, while in [29] Liu *et al.* applied Stackelberg game theory for relay selection in cellular networks. Zhang *et al.* [13] proposed a new framework for efficient resource management in cooperative cognitive radio network and formulated the problem of distributed relay selection and spectrum leasing as a Stackelberg game. In addition to [13], other excellent work on joint relay selection and spectrum management includes [30] and [31]. Zhao [30] investigated the power and spectrum allocation for cooperative relay in a three-node cognitive radio network. In [31], Ding *et al.* studied the cooperative diversity of three low-complexity relay selection strategies in spectrum-sharing networks. Different from the above work, we focus on interference-limited ad hoc networks with multiple concurrent sessions and multiple relay nodes.

Dynamic spectrum access has also been the focus of much recent attention, especially in the context of cognitive radio (CR) networks. For example, in [32], [33], Ekici *et al.* investigated cooperative spectrum sharing and leasing strategies between primary and secondary users. In [34], Chowdhury *et al.* proposed to adapt the classical TCP rate control to interact with the spectrum sensing and other lower-layer functionalities in secondary ad hoc networks. Hou *et al.* studied cross-layer resource allocation for capacity (or throughput) maximization in SINR-model-based [35] and MIMO-powered [36] CR networks. Different from these works, we consider dynamic spectrum access with cooperative relays.

Finally, cooperative communications have been also studied in conjunction with dynamic spectrum access. For example, [37]–[41] studied resource allocation in cellular cognitive radio (CR) networks with cooperative relays. In [42], Li *et al.* developed a centralized solution algorithm to maximize the minimum transmission rate among multiple source–destination pairs using cooperative communication in a cognitive radio network with orthogonal channels. In our previous work, the problem of distributed joint routing, relay selection, and spectrum allocation in interference-limited secondary ad hoc networks was investigated [43], where distributed algorithms were proposed based on a "backpressure" framework; and a centralized algorithm with optimality guarantee for joint relay selection and dynamic spectrum access in interference-limited video-streaming single-hop ad hoc networks was proposed in [44]. In this paper, we design a distributed algorithm to jointly allocate relays and spectrum in interference-limited infrastructure-less networks based on variational inequality theory and evaluate its performance by comparing it to an optimal centralized algorithm.

III. System Model and Problem Formulation

We consider a cognitive ad hoc wireless network, where a set $\mathcal{S}$ of communication sessions compete for spectrum resources. For each session, say $s \in \mathcal{S}$, a source–destination pair is identified. Each destination node is assumed to be reachable via one hop by its source node, while layer-3 routing in multihop networks will be investigated in our future work. Each source node can transmit to its destination node either using a direct link or through a cooperative relay. We refer to a link enhanced by a cooperative relay as a *cooperative link*. If a cooperative link is employed, the source node selects a node as relay from a set $\mathcal{R}$ of potential relay nodes. The available spectrum is divided into F channels; we denote the set of channels as $\mathcal{F}$. Each channel is potentially shared among different sessions, i.e., each session can be seen as an interferer to any other sessions, and each user dynamically selects the best channels to access to maximize its own utility.

Assumptions: We make the following two assumptions.

- *Simultaneous access to a channel.* Multiple sessions are allowed to access a channel at any given time. Therefore, they will cause interference to one another. To mitigate the effect of interference, dynamic spectrum access is employed, i.e., each session dynamically selects which channels to use and allocates transmission power on each channel, based on the channel quality of the underlying wireless link and on the interference measured at the destination node.
- *Single relay selection.* For the sake of simplicity, in this work we consider the single relay case, i.e., we assume that each relay node can be selected by at most one session, and one session can select at most one relay node. It is worth mentioning, however, that this work can be extended to the case of multiple relay selection.

There are several practical application scenarios that can be characterized using such a network model. For example, a snapshot of wireless sensor and ad hoc networks, where each currently active node transmits to its next-hop node using those nonactive nodes as potential relays; cellular networks with multiple mutually interfering uncoordinated cells, in each of which the scheduled user transmits to the base station via other nonscheduled users as relays. Similar models have also been adopted in existing literature, e.g., [16] and its multihop variation [17].

Cooperative Link Capacity Model: Denote the power allocation matrix for the source nodes as $\boldsymbol{P} = (P_s^f), s \in \mathcal{S}, f \in \mathcal{F}$, where P_s^f represents the transmission power for source node s on channel f. The power allocation matrix for the relay nodes is denoted with $\boldsymbol{Q} = (Q_r^f),\ r \in \mathcal{R}, f \in \mathcal{F}$, where Q_r^f is the transmission power for relay node r on channel f. Further denote the relay selection matrix as $\boldsymbol{\alpha} = (\alpha_r^s), r \in \mathcal{R}, s \in \mathcal{S}$, where $\alpha_r^s = 1$ if relay node r is selected by session s, and $\alpha_r^s = 0$ otherwise.

We let $C_s(\boldsymbol{P},\ \boldsymbol{Q},\ \boldsymbol{\alpha})$ represent the capacity available to session s, which can be expressed as

$$C_s(\boldsymbol{P},\ \boldsymbol{Q},\ \boldsymbol{\alpha}) = \underbrace{\left(1 - \sum_{r \in \mathcal{R}} \alpha_r^s\right) C_{\text{dir}}^s(\boldsymbol{P},\ \boldsymbol{Q})}_{\text{direct link}} + \underbrace{\sum_{r \in \mathcal{R}} \alpha_r^s C_{\text{cop}}^{s,r}(\boldsymbol{P},\ \boldsymbol{Q})}_{\text{cooperative link}} \quad (1)$$

where $C_{\text{dir}}^s(\boldsymbol{P},\ \boldsymbol{Q})$ represents the capacity available to session s if a direct link is used, and $C_{\text{cop}}^{s,r}(\boldsymbol{P},\ \boldsymbol{Q})$ represents the capacity of cooperative link if relay r is selected by session s. For a direct link, $C_{\text{dir}}^s(\boldsymbol{P},\ \boldsymbol{Q})$ can be expressed as

$$C_{\text{dir}}^s(\boldsymbol{P},\ \boldsymbol{Q}) = B \sum_{f \in \mathcal{F}} \log_2\left(1 + \frac{G_{s2d}^{s,s} P_s^f}{\delta_{s,f}^2 + I_s^f}\right) \quad (2)$$

where B is the bandwidth of each channel, $\delta_{s,f}^2$ represents the power of additive white Gaussian noise (AWGN) at destination node s on channel f, $G_{s2d}^{s,s}$ represents the average channel gain from source node s to destination node s, and I_s^f represents the interference measured at destination node s on channel f.

Different forwarding strategies can be employed for cooperative relaying, e.g., *amplify-and-forward* (AF) and *decode-and-forward* (DF) [3]. We assume that DF is used at each relay node, while AF will be addressed in our future work. Then, $C_{\text{cop}}^{s,r}(\boldsymbol{P},\ \boldsymbol{Q})$ can be expressed as [3], [16]

$$\begin{aligned} C_{\text{cop}}^{s,r}(\boldsymbol{P},\ \boldsymbol{Q}) &= \sum_{f \in \mathcal{F}} C_{\text{cop}}^{s,r,f}(\boldsymbol{P},\ \boldsymbol{Q}) \\ &= \frac{1}{2} \sum_{f \in \mathcal{F}} \min\left\{C_{s2r}^{s,r,f},\ C_{sr2d}^{s,r,f}\right\} \end{aligned} \quad (3)$$

where $C_{s2r}^{s,r,f}$ represents the capacity of link from source node s to relay node r on channel f, and $C_{sr2d}^{s,r,f}$ represents the capacity achieved through maximal ratio combining [3] on the two copies of the signal received by destination node s from source node s and relay node r on channel f. The coefficient $\frac{1}{2}$ in (3) indicates that the overall capacity for the cooperative link is averaged over two time-slots; as will be discussed later on in Section IV, the min operation in (3) results in a nonlinear nondifferentiable function, and this may impose significant challenges on algorithm design and theoretical analysis. Expressions for the two capacities are given by

$$C_{s2r}^{s,r,f}(\boldsymbol{P},\boldsymbol{Q}) = B \log_2\left(1 + \frac{G_{s2r}^{s,r} P_s^f}{\delta_{r,f}^2 + I_r^f}\right) \quad (4)$$

$$C_{sr2d}^{s,r,f}(\boldsymbol{P},\ \boldsymbol{Q}) = B \log_2\left(1 + \frac{G_{s2d}^{s,s} P_s^f + G_{r2d}^{r,s} Q_r^f}{\delta_{s,f}^2 + I_s^f}\right) \quad (5)$$

where $\delta_{r,f}^2$ represents Gaussian noise power at relay node r on channel f, $G_{s2r}^{s,r}$ and $G_{r2d}^{r,s}$ represent average channel gain from source node s to relay node r, and from relay node r to destination node s, respectively, and I_r^f represents interference measured at relay node r on channel f.

Interference Model: In (2), (4), and (5), the interference I_s^f and I_r^f depend not only on power allocation and relay selection at each individual node, but also on the network scheduling strategy of the whole network (i.e., the relative synchronization of transmission start times between different network communication links). To keep the model tractable, the interference at each receiver can be approximated in different ways. In the worst-case approximation, the assumption is that all source and active relay nodes cause interference in both time-slots. The average-based approximation considers instead the average effect of each interferer over the two time slots. Our experiments reveal that the average-based approximation models reality very well [44]. With this model, I_s^f can be expressed as

$$I_s^f = \sum_{w\in\mathcal{S}_{-s}} \left[\left(1 - \sum_{r\in\mathcal{R}} \alpha_r^w\right) G_{s2d}^{w,s} P_w^f + \frac{1}{2}\sum_{r\in\mathcal{R}} \alpha_r^w \left(G_{s2d}^{w,s} P_w^f + G_{r2d}^{r,s} Q_r^f\right)\right] \quad (6)$$

where $\mathcal{S}_{-s}$ represents the set of all sessions except session s. I_r^f has a similar expression.

Problem Formulation: We let U_s represent the utility function for session s and define it as

$$U_s = \log(C_s(\boldsymbol{P},\, \boldsymbol{Q},\, \boldsymbol{\alpha})) \quad (7)$$

where $C_s(\boldsymbol{P},\, \boldsymbol{Q},\, \boldsymbol{\alpha})$ is defined in (1) and a log-capacity utility function is considered to promote fairness among communication sessions. Then, the objective of our problem is to maximize a sum utility function of all communication sessions by selecting for each session: 1) which channels to allocate; 2) transmission power to be used on each selected channel; 3) whether to use a direct link or cooperative link; and 4) which relay node to select,[1] i.e.,

$$\begin{aligned}
\underset{\boldsymbol{P},\, \boldsymbol{Q},\, \boldsymbol{\alpha}}{\text{maximize}} \quad & U = \sum_{s\in\mathcal{S}} U_s(\boldsymbol{P},\, \boldsymbol{Q},\, \boldsymbol{\alpha}) && (8)\\
\text{subject to} \quad & \alpha_r^s \in \{0,\, 1\} \qquad \forall s\in\mathcal{S},\, \forall r\in\mathcal{R} && (9)\\
& \sum_{r\in\mathcal{R}} \alpha_r^s \le 1 \qquad \forall s\in\mathcal{S} && (10)\\
& \sum_{s\in\mathcal{S}} \alpha_r^s \le 1 \qquad \forall r\in\mathcal{R} && (11)\\
& P_s^f \ge 0 \qquad \forall s\in\mathcal{S},\, \forall f\in\mathcal{F} && (12)\\
& Q_r^f \ge 0 \qquad \forall r\in\mathcal{R},\, \forall f\in\mathcal{F} && (13)\\
& \sum_{f\in\mathcal{F}} P_s^f \le P_{\max}^s \qquad \forall s\in\mathcal{S} && (14)\\
& \sum_{f\in\mathcal{F}} Q_r^f \le Q_{\max}^r \qquad \forall r\in\mathcal{R} && (15)
\end{aligned}$$

where U represents sum utility, and $U_s(\boldsymbol{P},\, \boldsymbol{Q},\, \boldsymbol{\alpha})$ is defined in (7) (also denoted as U_s for conciseness). The expressions in (9)–(11) impose constraints on relay selection (at most one relay per session and one session per relay), while (12)–(15) impose constraints on power allocation and power budget for each source and relay node. Here, $P_{\max}^s$ and $Q_{\max}^r$ represent the maximum transmission power of source node s and relay node r, respectively.

In the problem formulated in (1)–(15), the expressions defined in (2)–(6) are nonlinear (and nonconvex) functions of the problem variables. Moreover, the relay selection variables α_r^s, $r \in \mathcal{R}, s \in S$, are constrained to take binary values (0 or 1). Therefore, the expression in (1) and consequently the objective function in (7) are both integral and nonconvex. This causes the problem to be a *Mixed-Integer and Non-Convex Problem* (MINCoP), which is in general NP-hard (i.e., no existing algorithm can solve an arbitrary MINCoP in polynomial time).

In the following sections, we first propose distributed algorithms designed to dynamically control node behavior based on localized best-response strategies. The original problem is decomposed into two separate problems, namely distributed relay selection with given power spectral profile and distributed spectrum allocation for a given relay selection. Then, we study the convergence of iterative algorithms based on iterative solutions of the two individual problems. We analyze the convergence and optimality of distributed algorithms for spectrum assignment and relay selection, based on notions from *variational inequality* (VI) theory. Finally, we design and study a centralized algorithm that provides an optimal solution to the problem with guaranteed convergence and that can be tuned to find a compromise between the desired precision and computation time. Results obtained through the centralized algorithm are employed to evaluate the "price of anarchy" of the proposed distributed algorithm.

IV. Distributed Algorithm

In this section, we propose a distributed algorithm for the problem formulated in Section III that is amenable to practical (distributed) implementation. The proposed distributed algorithm is designed to achieve the *Nash equilibrium* (NE) [45], which is a well-known concept from noncooperative game theory often used as a tool for designing distributed algorithms in complex wireless communication systems [11]–[13]. There are two important characteristics of an NE solution: 1) at any NE solution point, no user has incentives to deviate from the current transmission strategy unilaterally; and 2) each user's utility is maximized, given the transmission strategies of any other users. In this section, we study: 1) whether an NE solution point exists for our problem; 2) how to achieve such an NE solution point if it exists; and 3) the so-called price of anarchy, i.e., we compare the performance at NE solution point to the global optimal solution (obtained through a centralized algorithm).

The formulated MINCoP problem is nonlinear and nonconvex, which imposes major challenges to the NE analysis. Therefore, we decompose the original joint optimization problem into two separate subproblems, i.e., spectrum access and relay selection. For each subproblem, based on a powerful mathematical tool named *variational inequality* (VI) theory [15], [46], we demonstrate the existence of NE of the problem, design distributed solution algorithms and show that the designed algorithm converges, under certain conditions, to an NE of the subproblem. Finally, we evaluate performance of the distributed solution algorithms by comparing them to a newly designed centralized but globally optimal solution algorithm.

[1]In this work, it is assumed that the network state information, e.g., the channel gain between any two nodes, changes only slowly, and we optimize the power allocation and relay selection for each snapshot of the network.

A. Basics of VI Theory

For the reader's convenience, we provide definitions for a *variational inequality problem* and *Nash equilibrium problem*, respectively. Readers are referred to [15] for a detailed introduction to the relationship between them and [46] for a comprehensive overview of VI theory.

Definition 1 (Variational Inequality Problem): Given a closed and convex set $\mathcal{X} \in \mathbb{R}^n$ and a continuous mapping function $\boldsymbol{F} : \mathcal{X} \to \mathbb{R}^n$, the VI problem, denoted as $\text{VI}(\mathcal{X}, \boldsymbol{F})$, consists of finding a vector $\boldsymbol{x}^* \in \mathcal{X}$ (called a solution of the VI) such that [46]

$$(\boldsymbol{y} - \boldsymbol{x}^*)^T \boldsymbol{F}(\boldsymbol{x}^*) \leq 0 \qquad \forall \boldsymbol{y} \in \mathcal{X}. \tag{16}$$

Definition 2 (Nash Equilibrium Problem): Assume there are Q players each controlling a variable $\boldsymbol{x}_i \in \mathcal{Q}_i$. Denote $\boldsymbol{x}$ as the vector of all variables $\boldsymbol{x} \triangleq (x_i, \ldots, x_Q)$, and let $\boldsymbol{x}_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_Q)$ represent the vector of all player variables except that of player i. Each player i is also associated with a utility function $f_i(\boldsymbol{x}_i, \boldsymbol{x}_{-i})$. Define the Cartesian product of all $\mathcal{Q}_i$ as $\mathcal{Q} \triangleq \prod_{i=1}^{Q} \mathcal{Q}_i$, the vector of utility functions as $\boldsymbol{f} = (f_1, \ldots, f_Q)$. Then, a Nash equilibrium problem, denoted as $\text{NE}(\mathcal{Q}, \boldsymbol{f})$, consists of finding $\boldsymbol{x}^* = (\boldsymbol{x}_1^*, \ldots, \boldsymbol{x}_Q^*)$ (called Nash equilibrium solution), such that each player i's utility function $f_i(\boldsymbol{x}_i, \boldsymbol{x}_{-i}^*)$ is maximized, i.e., [45]

$$\boldsymbol{x}_i^* = \arg\max_{\boldsymbol{x}_i \in \mathcal{Q}_i} f_i(\boldsymbol{x}_i, \boldsymbol{x}_{-i}^*) \qquad \forall i. \tag{17}$$

Given a Nash equilibrium problem $\text{NE}(\mathcal{Q}, \boldsymbol{f})$, assume that for each player i: 1) the strategy set $\mathcal{Q}_i$ is closed and convex; and 2) the utility function $f_i(\boldsymbol{x}_i, \boldsymbol{x}_{-i})$ is continuously differentiable with respect to $\boldsymbol{x}_i$ in $\mathcal{Q}_i$. Then, the Nash equilibrium problem $\text{NE}(\mathcal{Q}, \boldsymbol{f})$ is equivalent to the $\text{VI}(\mathcal{Q}, \boldsymbol{F})$, where $\boldsymbol{F} \triangleq (\nabla_{\boldsymbol{x}_i} f_i(\boldsymbol{x}))_{i=1}^{Q}$ [15]. Hence, to achieve an NE solution for $\text{NE}(\mathcal{Q}, \boldsymbol{f})$, we only need to find a VI solution for $\text{VI}(\mathcal{Q}, \boldsymbol{F})$.

It should be pointed out that to take advantage of the well-developed theoretical results and of the existing distributed algorithms of the VI theory, a condition required by VI theory is that the mapping function $\boldsymbol{F}$ be at least componentwise strongly monotonic [46]. Recall that the utility function in our problem has a rather complex expression, which makes the monotonicity analysis very hard. To address this challenge, we decompose the problem formulated in Section III into two individual problems: 1) distributed power allocation, for a given cooperative relaying strategy; and 2) distributed relay selection, for a given spectrum profile. Then, we study the problem of joint relay selection and power allocation based on the two individual problems.

B. Distributed Spectrum Management (DSM) by Power Allocation

1) Game Theory Formulation: A problem of DSM with given cooperative relaying strategy can be naturally formulated as a game, in which each communication session can be seen as a player, and each player tries to maximize its utility function defined in (7) by adjusting the transmission power over the available frequencies for its source node and corresponding relay node if cooperative relaying is employed. Hereafter, we use the terms of "communication session" and "player" interchangeably.

Assume relay node r is selected by session s, and denote the vector of power allocation for source node s and relay node r as $\boldsymbol{P}_s = (P_s^f),\ f \in \mathcal{F}$ and $\boldsymbol{Q}_r = (Q_r^f),\ f \in \mathcal{F}$, respectively. Then, any $\boldsymbol{P}_s$ or $\boldsymbol{Q}_r$ is feasible if it satisfies the constraints in (12) and (14), or the constraints in (13) and (15), respectively. We let $\mathcal{P}_s$ and $\mathcal{Q}_r$ represent the set of all feasible $\boldsymbol{P}_s$ and $\boldsymbol{Q}_r$, and denote the vector of power allocation for session s as $\boldsymbol{x}_s = (\boldsymbol{P}_s,\ \boldsymbol{Q}_r)$. Then, the set of transmission strategies for session s, denoted as $\mathcal{X}_s$, can be defined as the Cartesian product of $\mathcal{P}_s$ and $\mathcal{Q}_s$, i.e., $\mathcal{X}_s \triangleq \mathcal{P}_s \times \mathcal{Q}_r$. If session s uses only direct link, the set of transmission strategies simply reduces to $\mathcal{X}_s = \mathcal{P}_s$.

The utility function for communication session $s \in \mathcal{S}$ is defined in (7), where $C_s(\boldsymbol{P},\ \boldsymbol{Q})$ is defined in (2) if only direct link is used, and in (3) otherwise. Define $\boldsymbol{P}_{-s} = (\boldsymbol{P}_w),\ w \in \mathcal{S},\ w \neq s$, and $\boldsymbol{Q}_{-r} = (\boldsymbol{Q}_q),\ q \in \mathcal{R},\ q \neq r$, as the vectors of power allocation for all other source and relay nodes. Then, with given fixed $\boldsymbol{\alpha}$ and $\boldsymbol{x}_{-s} = (\boldsymbol{P}_{-s}, \boldsymbol{Q}_{-r})$, the utility function of session s can be rewritten as

$$\begin{aligned} U_s(\boldsymbol{x}_s, \boldsymbol{x}_{-s}) &= \log(C_s(\boldsymbol{x}_s,\ \boldsymbol{x}_{-s})) \\ &= \log(C_s(\boldsymbol{P}_s,\ \boldsymbol{Q}_r,\ \boldsymbol{P}_{-s},\ \boldsymbol{Q}_{-r})) \end{aligned} \tag{18}$$

where $C_s(\boldsymbol{P}_s,\ \boldsymbol{Q}_r,\ \boldsymbol{P}_{-s},\ \boldsymbol{Q}_{-r})$ is defined in the same way as $C_s(\boldsymbol{P},\ \boldsymbol{Q})$ with $\boldsymbol{P} = (\boldsymbol{P}_s, \boldsymbol{P}_{-s})$ and $\boldsymbol{Q} = (\boldsymbol{Q}_r, \boldsymbol{Q}_{-r})$.

According to Definition 2, the DSM problem can be modeled as a game $\text{NE}(\mathcal{X}, \boldsymbol{U})$, where $\mathcal{X} = \prod_{s \in \mathcal{S}} \mathcal{X}_s$ and $\boldsymbol{U} = (U_s)_{s \in \mathcal{S}}$.

2) VI Formulation: Recall from Section IV-A that a given NE problem is equivalent to a VI if the utility function for each player is defined in a closed and convex domain set and is continuously differentiable. In the DSM problem formulated above, the domain set of $\mathcal{X}$ is closed and convex since it is defined by a set of linear constraints in (12)–(15). However, the utility function for each player is certainly not continuously differentiable. This is because if cooperative relaying is employed by a communication session, the capacity of the cooperative link is defined in (3), where the minimum operation leads to a nonsmooth function. Hence, the resulting utility function is not continuously differentiable.

To facilitate the analysis, we can approximate the cooperative link capacity in (3) using a continuously differentiable function, denoted as $\widehat{C}_{\text{cop}}^{s,r,f}$, constructed based on ℓ_P-norm function as follows:

$$\begin{aligned} \widehat{C}_{\text{cop}}^{s,r,f} &= \ell_P^{-1}((C_{s2r}^{s,r,f})^{-1},\ (C_{sr2d}^{s,r,f})^{-1}) \\ &= \left\{ \left[\left(\frac{1}{C_{s2r}^{s,r,f}} \right)^P + \left(\frac{1}{C_{sr2d}^{s,r,f}} \right)^P \right]^{\frac{1}{P}} \right\}^{-1} \end{aligned} \tag{19}$$

where $C_{s2r}^{s,r,f}$ and $C_{sr2d}^{s,r,f}$ are defined in (4) and (5), respectively. The ℓ_P-norm of a vector with large value of parameter P emphasizes the larger element in the vector [47], hence it also emphasizes the element with the smallest inverse. The resulting approximation function is continuously differentiable in its domain, and the original function in (3) can be approximated with arbitrary precision by adjusting the value of parameter P. When $P \to \infty$, we have $\widehat{C}_{\text{cop}}^{s,r,f} \to C_{\text{cop}}^{s,r,f}$.

Based on the above approximation, the utility function for each player becomes continuously differentiable, and hence we can rewrite the NE problem in Section IV-B.1 as a VI problem $\text{VI}(\mathcal{X}, \boldsymbol{F})$, with $\boldsymbol{F} = (\nabla_{\boldsymbol{x}_s} U_s),\ s \in \mathcal{S}$.

3) VI Solution and Distributed Algorithm: After obtaining a VI formulation of the problem, we can study whether a VI solution (which is also an NE solution) exists for the problem, and if it exists, how to achieve it in a distributed fashion. Before developing a distributed algorithm, we give the following two lemmas about the domain set $\mathcal{X}$ and the utility function for each player $U_s(\boldsymbol{x}_s, \boldsymbol{x}_{-s})$, respectively.

Lemma 1: There exists at least one solution for VI$(\mathcal{X}, \boldsymbol{F})$.

Proof: The domain set $\mathcal{X}$ is closed and convex, and the mapping function $\boldsymbol{F}$ is continuous. According to the existence theorem in [46], there exists at least one VI solution. ■

Lemma 2: Assume that relay node r is selected by communication session s for cooperative relaying. Then, the utility function $U_s(\boldsymbol{x}_s, \boldsymbol{x}_{-s})$ is a strongly concave function if it satisfies the condition that the signal-to-interference-plus-noise ratio (SINR) on each channel for the link between source node s and destination node s, source node s and relay node r, relay node r and destination node s, is greater than $e-1$, where e is the base of natural logarithm.

Proof: A strong concave function is a function that is concave and its derivative does not approach zero in its domain [46]. For our case, the approximation function $\widehat{C}_{\text{cop}}^{s,r,f}$ in (19) is a monotonically increasing function of the transmission power. The domain set $\mathcal{X}$ in the VI problem VI$(\mathcal{X}, \boldsymbol{F})$ is bounded. Therefore, the derivative of $\widehat{C}_{\text{cop}}^{s,r,f}$ with respect to the transmission power for source or relay node on each channel is positive and cannot be arbitrarily small. Hence, to prove that $\widehat{C}_{\text{cop}}^{s,r,f}$ is strong concave, we only need to show that it is a concave function. This can be proven by showing that $\widehat{C}_{\text{cop}}^{s,r,f}$ is concave within its domain $\mathcal{X}_s$ restricted to an arbitrary line [47]. ■

The condition in Lemma 2 is a sufficient condition for the utility function $U_s(\boldsymbol{x}_s, \boldsymbol{x}_{-s})$ to be strongly concave, and requires that the SINR on each channel is not too low[2] for a cooperative link. In practice, a communication session will never allocate much transmission power to a channel with very bad channel quality. Moreover, if all available channels have very bad quality, then even minimum quality-of-service (QoS) requirements cannot be guaranteed. In this case, the communication session should be denied access to the wireless network. Alternatively, the session should increase its maximum transmission power or select another routing path to avoid channels with poor quality. Therefore, with no loss of generality for all practical purposes, we assume that the condition in Lemma 2 can be satisfied.

Next, we propose an iterative algorithm of dynamic spectrum management which is based on the local best-response of each communication session, and analyze the necessary and sufficient conditions for the algorithm to converge to a VI solution of VI$(\mathcal{X}, \boldsymbol{F})$.

[2]A value of $e-1 \approx 1.71828$ for SINR implies that the received signal power is comparable to the sum of noise and interference—which corresponds to very poor channel quality.

Algorithm 1: Gauss–Seidel Best-Response Algorithm for DSM

Step 1: Initialize to any feasible power allocation $\boldsymbol{x}^{(0)} = (\boldsymbol{x}_s^{(0)})_{s\in\mathcal{S}}$ that satisfies the constraints in (12)–(15), set iteration index $n = 0$.

Step 2: For $s = 1, \ldots, S$, calculate $\boldsymbol{x}_s^{(n+1)}$ by solving

$$\begin{aligned} \underset{\boldsymbol{x}_s}{\text{maximize}} \quad & U_s(\boldsymbol{x}_1^{(n+1)}, \ldots, \boldsymbol{x}_{s-1}^{(n+1)}, \boldsymbol{x}_s, \boldsymbol{x}_{s+1}^{(n)} \ldots, \boldsymbol{x}_S^{(n)}) \\ \text{s.t.} \quad & \boldsymbol{x}_s \in \mathcal{X}_s. \end{aligned} \tag{20}$$

Step 3: Set $\boldsymbol{x}^{(n+1)} = (\boldsymbol{x}_s^{(n+1)})_{s=1}^{S}$ and set $n \leftarrow n+1$.

Step 4: If $\boldsymbol{x}^{(n+1)}$ is a VI solution of the problem in (16), terminate, and go to step 2 otherwise.

We give the Gauss–Seidel implementation of the local best-response based algorithm in Algorithm 1, where S represents the number of communication sessions in $\mathcal{S}$. In Algorithm 1, a feasible initial power allocation means that the power level at a subchannel cannot be too low. Otherwise, there would be no guarantee of convergence according to Lemma 2. An infeasible initial power allocation can be avoided by artificially increasing power values that are too low, while an infeasible power allocation during iterations in Algorithm 1 can be avoided by allocating zero power to a subchannel with poor quality (i.e., do not select poor subchannels for transmission).

Theorem 1: Given the VI problem VI$(\mathcal{X}, \boldsymbol{F})$ formulated in Section IV-B.2, a Gauss–Seidel scheme based on the local best-response of each communication session converges to a VI solution if the following conditions hold.

i) Lemmas 1 and 2 hold.

ii) Any two sessions in $\mathcal{S}$ are located sufficiently far away from each other, or equivalently the wireless channel gains between any two sessions are sufficiently low, i.e., the expression

$$\inf_{\boldsymbol{x}\in\mathcal{X}} \lambda_s(\boldsymbol{x}) > \sup_{\boldsymbol{x}\in\mathcal{X}} \sum_{g\in\mathcal{S}/s} \widehat{\beta}_{sg}^{\max}(\boldsymbol{x}) \qquad \forall s \in \mathcal{S} \tag{21}$$

is satisfied, where $\boldsymbol{x} = \left((P_s^f)_{s\in\mathcal{S}}^{f\in\mathcal{F}}, (Q_r^f)_{r\in\mathcal{R}}^{f\in\mathcal{F}}\right)$, and $\lambda_s(\boldsymbol{x})$ and $\widehat{\beta}_{sg}^{\max}(\boldsymbol{x})$ are given in (22) and (23) at the bottom of the page, respectively, with $|\mathcal{F}|$ representing the cardinality of the spectrum set $\mathcal{F}$, r_s and r_g denoting the relay

$$\lambda_s(\boldsymbol{x}) = \min_{f\in\mathcal{F}} \frac{1}{2}\left|\left(\frac{\partial^2 \widehat{C}_{\text{cop}}^{s,r_s,f}}{\partial P_s^f \partial P_s^f} + \frac{\partial^2 \widehat{C}_{\text{cop}}^{s,r_s,f}}{\partial Q_{r_s}^f \partial Q_{r_s}^f}\right) + \sqrt{\left(\frac{\partial^2 \widehat{C}_{\text{cop}}^{s,r_s,f}}{\partial P_s^f \partial P_s^f} + \frac{\partial^2 \widehat{C}_{\text{cop}}^{s,r_s,f}}{\partial Q_{r_s}^f \partial Q_{r_s}^f}\right)^2 - 4\frac{\partial^2 \widehat{C}_{\text{cop}}^{s,r_s,f}}{\partial P_s^f \partial Q_{r_s}^f}\frac{\partial^2 \widehat{C}_{\text{cop}}^{s,r_s,f}}{\partial Q_{r_s}^f \partial P_s^f}}\right| \tag{22}$$

$$\widehat{\beta}_{sg}^{\max}(\boldsymbol{x}) = 2|\mathcal{F}| \max_{f\in\mathcal{F}} \max\left(\left|\frac{\partial^2 \widehat{C}_{\text{cop}}^{g,r_g,f}}{\partial P_g^f \partial P_s^f}\right|, \left|\frac{\partial^2 \widehat{C}_{\text{cop}}^{g,r_g,f}}{\partial P_g^f \partial Q_{r_s}^f}\right|, \left|\frac{\partial^2 \widehat{C}_{\text{cop}}^{g,r_g,f}}{\partial Q_{r_g}^f \partial P_s^f}\right|, \left|\frac{\partial^2 \widehat{C}_{\text{cop}}^{g,r_g,f}}{\partial Q_{r_g}^f \partial Q_{r_s}^f}\right|\right) \tag{23}$$

nodes selected by sessions $s, g \in \mathcal{S}$, respectively, and $\widehat{C}_{\text{cop}}^{s,r_s,f}$ and $\widehat{C}_{\text{cop}}^{s,r_g,f}$ defined in (19).

Proof: See Appendix-A please. ■

C. Distributed Relay Selection (DRS)

1) Game-Theoretic Formulation: The problem of relay selection with given fixed spectrum profile can also be formulated as a game, in which each communication session selects its best relay node in a competitive fashion to maximize its own utility function in (7). Recall that in Section III we assumed that only one single relay is selected by a communication session. Therefore, competition occurs if a relay node is the best relay for more than one session. Moreover, as shown in (6), the transmission strategy of a session, i.e., using only direct link or cooperative link, also affects the interference caused by the session to the other sessions.

Denote the vector of relay selection variables for communication session $s \in \mathcal{S}$ as $\boldsymbol{\alpha}_s = (\alpha_r^s)$, $r \in \mathcal{R}$, and the vector of relay selection variables for all other communication sessions except s as $\boldsymbol{\alpha}_{-s} = (\alpha_r^w)$, $r \in \mathcal{R}$, $w \in \mathcal{S}$, $w \neq s$. Then, $\boldsymbol{\alpha} = (\boldsymbol{\alpha}_s, \boldsymbol{\alpha}_{-s})$. We let Φ_s represent the set of all possible $\boldsymbol{\alpha}_s$, and Φ represent the set of all possible $\boldsymbol{\alpha}$. Given a fixed spectrum profile and relay selection strategies for all other communications, the utility function for session s in (7) can be rewritten as $U_s(\boldsymbol{\alpha}_s, \boldsymbol{\alpha}_{-s})$. Denote the vector of all utility functions as $\boldsymbol{U} = (U_s)$, $s \in \mathcal{S}$. Then, the Nash equilibrium problem of relay selection can be formulated as $\text{NE}(\Phi, \boldsymbol{U})$.

Each individual domain set Φ_s is described by constraints in (9) and (11), while the overall domain set Φ is described by constraints in (9)–(11). Since all communication sessions are coupled through constraint (10), Φ cannot be written in the form of the Cartesian product of Φ_s. Given $\boldsymbol{\alpha}_{-s}$, the set Φ_s is a function of $\boldsymbol{\alpha}_{-s}$, i.e., $\Phi_s = \Phi_s(\boldsymbol{\alpha}_{-s})$. Hence, the Nash equilibrium problem $\text{NE}(\Phi, \boldsymbol{U})$ is not a standard NE problem as defined in Definition 2 with respect to the domain set. In this case, the $\text{NE}(\Phi, \boldsymbol{U})$ is called a generalized Nash equilibrium (GNE) problem, denoted as $\text{GNE}(\Phi, \boldsymbol{U})$. We propose a penalization-based algorithm to transform the $\text{GNE}(\Phi, \boldsymbol{U})$ into a series of standard NE problems, which can then be analyzed and solved using the existing VI theory.

2) VI Reformulation: In the formulated GNE problem, the domain set Φ is closed and convex. Moreover, with given spectrum profile and fixed relay selection strategy for all communication sessions except s, the capacity for session s in (2) and (3) is fixed. Hence, $U_s(\boldsymbol{\alpha}_s, \boldsymbol{\alpha}_{-s})$ becomes a function of $\boldsymbol{\alpha}_s$ only. To cast the GNE problem into a VI problem to make the theoretical analysis easier, we still need to relax the integer problem (due to the binary relay selection variables) to a continuous one. To this end, in the following discussion we relax the binary requirement and let each α_r^s, $r \in \mathcal{R}$, $s \in \mathcal{S}$ be real. Effects of the relaxation will be analyzed later together with the proposed distributed algorithm. Then, the utility function $U_s(\boldsymbol{\alpha}_s, \boldsymbol{\alpha}_{-s})$ becomes continuously differentiable, and $\text{GNE}(\Phi, \boldsymbol{U})$ can be reformulated as a VI problem with mapping function $\boldsymbol{F} = (F_s)_{s \in \mathcal{S}}$, where $F_s = \nabla_{\boldsymbol{\alpha}_s} U_s$. Corresponding to a GNE, the VI problem with coupled domain set Φ is called a quasi-VI (QVI) problem, denoted as $\text{QVI}(\Phi, \boldsymbol{F})$. Before developing distributed relay selection algorithm based on the QVI reformulation of the GNE problem, we first give the following lemma.

Lemma 3: There exists at least one VI solution (also Nash equlibrium) for $\text{QVI}(\Phi, \boldsymbol{F})$.

Proof: For a QVI problem with closed and convex domain set and continuous utility function, there exists at least one VI solution solving the QVI problem [46]. ■

3) Distributed DRS Algorithm: Each communication session, say s, locally decides its optimal relay selection strategy $\boldsymbol{\alpha}_s$ for a given $\boldsymbol{\alpha}_{-s}$. Since the interference measured at each destination and corresponding relay nodes are affected by the relay selection strategies of all other sessions, an update of relay selection for any communication session will trigger update of relay selection for all other sessions. More importantly, the set of possible relay selection strategies for each communication session also changes, i.e., $\boldsymbol{\alpha}_s$ is a function of $\boldsymbol{\alpha}_{-s}$.

A natural way to address this case with coupled domain sets is to set a price for each relay node and make each communication session pay a price to it [15]. Then, each relay node updates its price based on the relay selection strategies of all sessions. If more than one session selects the same relay node, then the price for the relay node is increased. Otherwise, the relay node keeps its price unchanged. However, a main concern of the price-based algorithm is how to guarantee that the algorithm converges to a VI solution, which is also an NE solution for the NE problem. We propose to design the price-based algorithm using a penalized version of the original utility function for each communication session such that the resulting algorithm can be proven to converge to a VI solution of the DRS problem.

The proposed algorithm converges to a VI solution iteratively. At iteration k, communication session s has a penalized version of the utility function $U_s(\boldsymbol{\alpha}_s, \boldsymbol{\alpha}_{-s})$, denoted as $\widehat{U}_s(\boldsymbol{\alpha}_s, \boldsymbol{\alpha}_{-s})$, as follows:

$$\widehat{U}_s(\boldsymbol{\alpha}_s, \boldsymbol{\alpha}_{-s}) = U_s(\boldsymbol{\alpha}_s, \boldsymbol{\alpha}_{-s}) \underbrace{-\frac{1}{2\rho_k} \sum_{r \in \mathcal{R}} \left(\max\left(0, u_k^r + \rho_k \left(\sum_{s \in \mathcal{S}} \alpha_r^s - 1 \right)\right)\right)^2}_{\text{Penalization}} \tag{24}$$

where the penalization term is a function of relay selection variables $\alpha_r^s, r \in \mathcal{R}, s \in \mathcal{S}$, with ρ_k, u_k^r being iteration parameters (we discuss later in this section how to choose values for them).[3]

In (24), each player's utility is penalized by subtracting a value, which is zero if the constraint in (11) is not violated, and is positive otherwise. $\{\rho_k\}$, $k = 0, 1, \ldots$, is a sequence of positive scalars and satisfies $\rho_k < \rho_{k+1}$ and $\rho_k \to \infty$ as $k \to \infty$. $\{\boldsymbol{u}_k\}$, $k = 1, 2, \ldots$, is a bounded sequence of vectors with $\boldsymbol{u}_k = (u_k^r)$, $r \in \mathcal{R}$. We will see later that u_k^r is used as the price for relay node r at iteration k, while ρ_k is employed as the stepsize based on which each relay node updates its price.

Based on the penalized function $\widehat{U}_s(\boldsymbol{\alpha}_s, \boldsymbol{\alpha}_{-s})$, we can construct a new VI problem $\text{VI}(\widehat{\Phi}, \widehat{\boldsymbol{F}})$, where $\widehat{\Phi}$ is the Cartesian product of each individual domain set Φ_s, $s \in \mathcal{S}$, and $\widehat{\boldsymbol{F}} = (\nabla_{\alpha_s} \widehat{U}_s)$, $s \in \mathcal{S}$. Moreover, we have that Lemma 4 holds true for each $\nabla_{\alpha_s} \widehat{U}_s$.

[3]The core idea of the penalization algorithm is to penalize the coupled relay selection constraint in (11) following an idea similar to the augmented Lagrangian approach, where, according to [48], the quadratic penalty has been preferred for several reasons. Among these, there is little extra computational cost, the parameter ρ_k need not go to zero. While different penalization methods can also be designed, this is however outside the scope of this work.

Algorithm 2: Penalization-Based Algorithm for DRS

Step 1: Initialize $\rho_0 = 0$, $\boldsymbol{u}_0 = 0$, and set $k = 0$.
Step 2: Calculate VI solution $\boldsymbol{\alpha}_k$ by solving VI problem VI($\widehat{\Phi}$, $\widehat{\boldsymbol{U}}$), in which each communication session solves

$$\begin{aligned} \underset{\boldsymbol{\alpha}_s}{\text{maximize}} \quad & \widehat{U}_s(\boldsymbol{\alpha}_s,\ \boldsymbol{\alpha}_{-s}) \text{ in (24)} \\ \text{s.t.} \quad & \boldsymbol{\alpha}_s \in \Phi_s. \end{aligned} \tag{25}$$

Step 3: Update ρ_k and $\boldsymbol{u}_k$ according to (26) and (27), respectively. Set $k \leftarrow k+1$.
Step 4: If condition (10) is satisfied for each relay node $r \in \mathcal{R}$, stop. Otherwise, go to Step 2.

Lemma 4: $\nabla_{\alpha_s}\widehat{U}_s$ is a strongly monotonic function of $\boldsymbol{\alpha}_s$ for each $s \in \mathcal{S}$.

Proof: We only need to show that the penalized utility function $\widehat{U}_s$ is strongly concave with respect to $\boldsymbol{\alpha}_s$ [15]. Then, it is sufficient to show that both $U_s(\boldsymbol{\alpha}_s,\ \boldsymbol{\alpha}_{-s})$ and the penalization item in (24) are strongly concave. This can be proven based on the definition of strong concavity [15] and on composition rules that preserve concavity [47, Sec. 3.2], i.e.: 1) componentwise maximum of two affine functions is convex; 2) the square of a convex positive function is convex; and 3) the opposite of a convex function is concave. ■

Based on Lemma 4 and Theorem 1, a VI solution for VI($\widehat{\Phi}$, $\widehat{\boldsymbol{U}}$) can be calculated through a best-response based algorithm similar to Algorithm 1. Denote the VI solution obtained at iteration k with $\boldsymbol{\alpha}_k = (\boldsymbol{\alpha}_s^k)$, where $\boldsymbol{\alpha}_s^k = (\alpha_{s,r}^k)$, $s \in \mathcal{S}$, $r \in \mathcal{R}$. Then, we can update ρ_k and $\boldsymbol{u}_k$ as follows:

$$\rho_{k+1} = \rho_k + \Delta\rho \tag{26}$$

$$u_{k+1}^r = \max\left(0,\ u_k^r + \rho_k\left(\sum_{s\in\mathcal{S}} \alpha_{s,r}^k - 1\right)\right) \tag{27}$$

where $\Delta\rho$ is any fixed positive constant. The proposed algorithm is summarized in Algorithm 2, and for the algorithm we have that Lemma 5 holds true.

Lemma 5: The proposed penalized algorithm always converges to a VI solution for QVI($\Phi, \boldsymbol{F}$), which is also an NE solution for GNE($\Phi, \boldsymbol{U}$).

Proof: Readers are referred to Appendix-B for a proof of the lemma. ■

D. Joint Spectrum Management and Relay Selection

We have so far studied and solved two problems: 1) power allocation with given fixed relay selection, and 2) relay selection with given spectrum profile. Based on the above analysis, next we develop two algorithms for joint spectrum management and relay selection, one based on decomposing the original joint optimization problem formulated in Section III and the other based on a transformation of the problem.

Decomposition-Based Solution Algorithm: This algorithm solves the original by solving the above two individual problems iteratively. First, the power allocation vectors $\boldsymbol{P}$ and $\boldsymbol{Q}$ are initialized based on any feasible power allocation, e.g., equal power allocation over all channels for each source and relay node. Then, based on $\boldsymbol{P}$ and $\boldsymbol{Q}$, a VI solution of relay selection $\boldsymbol{\alpha}$ can be obtained through Algorithm 2. Note that in Section IV-C, we assume that each α_r^s, $r \in \mathcal{R}$, $s \in \mathcal{S}$ is real. Hence, the VI solution $\boldsymbol{\alpha}$ might not be feasible for the original problem formulated in Section III, where each α_r^s takes only integer values of 0 or 1. To get a feasible $\boldsymbol{\alpha}$, we perform a $round(\cdot)$ operation to each α_r^s as follows:

$$\widehat{\alpha}_r^s = round(\alpha_r^s), \qquad r \in \mathcal{R},\ s \in \mathcal{S}. \tag{28}$$

If rounding gives an unfeasible solution, e.g., $\widehat{\alpha}_r^s = \widehat{\alpha}_r^w = 1$, $s, w \in \mathcal{S}$, $s \neq w$, the relay is assigned to session s if $\alpha_r^s \geq \alpha_r^w$, and assigned to session w otherwise. Denote the resulting vector of feasible relay selection as $\widehat{\boldsymbol{\alpha}}$. Then, with given $\widehat{\boldsymbol{\alpha}}$, we solve the problem of power allocation using Algorithm 1. The above iteration continues until the objective function in (7), i.e., sum utility of all communication sessions, does not change any more or the maximum number of iterations is reached.

It is worth pointing out that while this algorithm is in general not guaranteed to converge to the global or any local optimum of the original optimization problem, its performance is typically excellent. In Section VI, we evaluate its performance by comparing it to a newly designed centralized but globally optimal solution algorithm. A potential research direction is to design distributed solution algorithms based on the recent results [49], as in [50]; the resulting distributed solution algorithms are however guaranteed to converge to a stationary point of the social optimization problem only; to the best of our knowledge, it is still an open problem to design distributed solution algorithms with theoretically guaranteed convergence to a Nash equilibrium (if there exists any), given arbitrarily complex utility expressions.

Transformation-Based Solution Algorithm: The core idea is to transform the original joint optimization problem into an equivalent relay selection problem, and then solve the resulting DRS problem using again Algorithm 2.

Consider the joint power allocation and relay selection for a single session $s \in \mathcal{S}$, given transmission profiles of all other sessions $\boldsymbol{P}_{-s}$, $\boldsymbol{Q}_{-s} \triangleq (\boldsymbol{Q}_{r_w})_{w\in\mathcal{S}/s}$, and $\boldsymbol{\alpha}_{-s}$. The objective of session s is then to maximize its achievable utility $U_s(\boldsymbol{P}_s, \boldsymbol{Q}_s, \boldsymbol{\alpha}_s, \boldsymbol{P}_{-s}, \boldsymbol{Q}_{-s}, \boldsymbol{\alpha}_{-s})$ defined through (7), with $\boldsymbol{Q}_s \triangleq (\boldsymbol{Q}_r)_{r\in\mathcal{R}/\mathcal{R}_{-s}}$ and $\mathcal{R}_{-s} \triangleq \{r_w, w \in \mathcal{S}/s\}$. This again results in a GNE game, for which the existence of NE can also be established; the proof is similar to Lemma 3 and is hence omitted. To optimize the utility for each individual session, we discuss a two-step solution algorithm, as follows.

i) For each candidate relay node $r \in \mathcal{R}$, assuming that it is selected by session s, compute the optimal power allocation $\boldsymbol{P}_s$ and $\boldsymbol{Q}_r$, by solving a convex optimization problem (20). Denote the resulting maximal cooperative link capacity as $C_{\text{cop}}^{s,r,*}$. If session s uses direct transmission only, denote the maximal link capacity as $C_{\text{dir}}^{s,*}$.
ii) Session s decides the optimal relay selection strategy by maximizing

$$U_s \triangleq \log\left((1 - \sum_{r\in\mathcal{R}} \alpha_r^s) C_{\text{dir}}^{s,*} + \sum_{r\in\mathcal{R}} \alpha_r^s C_{\text{cop}}^{s,r,*}\right) \tag{29}$$

which is a convex optimization problem.

Then, the joint optimization problem for each session reduces to a relay selection problem, and the multisession optimization problem can be casted as in Section IV-C.1 into a relay selection game. A similar penalization-based algorithm as in Algorithm 2 can again be designed. It is worth pointing out that, although the optimization in step ii) is convex, the two-step overall optimization is however nonconvex, implying that a penalization-based algorithm similar to Algorithm 2 may not be guaranteed to converge any more in this case.

E. Implementation Issues

The proposed algorithm can be directly applied to a scenario with multiple coexisting preestablished source–destination pairs. In addition, it can be used to optimally control resource allocation for an independent set of transmissions with primary interference constraints (i.e., no transmitters and receivers in common) periodically scheduled by a separate scheduling algorithm, where idle nodes can be used as potential relays.

There are several potential alternative implementations for the proposed algorithm. To allow each source and potential relay to select the best subchannels, the power level of noise plus interference on each subchannel need to be measured at the destination and relay and then fed back to the source. Alternatively, they can be estimated at the source based on control information overheard on a control channel. This can be carried out through a cooperative MAC protocol. A good example of such protocols is the CoCogMAC proposed in [43], which uses a three-way handshake to exchange Request-to-Send (RTS), Clear-to- Send (CTS), and Relay-Ready-to-Relay (RTR) frames among the source, destination, and the selected relay to inform the source (and neighboring devices) of transmitted power chosen on each channel. For relay selection, each destination node needs to estimate the quality of all subchannels from itself to the corresponding source, while each relay needs to measure the channel quality from itself to each source and to each destination. This can be also carried out through a protocol similar to CoCogMAC. Additionally, to implement the pricing strategy in relay selection, each potential relay needs to periodically broadcast a "price" frame to claim its price on a common control channel that can be implemented out of band, in a time-sharing or in a code-division fashion.

V. Centralized Solution Algorithm

Denote U^* as the optimal value of the problem in (8)–(15). Then, the centralized algorithm is designed to search for a solution U that satisfies

$$U \geq \epsilon U^* \tag{30}$$

where $0 < \epsilon < 1$ represents a predefined optimality precision that can be set as close to 1 as we wish at the cost of computational complexity.

A. Algorithm Overview

We design the centralized solution algorithm based on the *branch-and-bound* (B&B) framework [47], [51] and on *convexification* of the original nonconvex problems formulated in Section III to iteratively search for the optimal solution. At each iteration, the algorithm maintains a global upper bound UP_{glb} and a global lower bound LR_{glb} on the social objective U in (8) such that

$$\text{LR}_{\text{glb}} \leq U^* \leq \text{UP}_{\text{glb}}. \tag{31}$$

Let Υ represent the domain set, i.e., the feasible set of the original problem defined through (8)–(15). Then, the proposed algorithm maintains a set of subdomains $\widetilde{\Upsilon} = \{\Upsilon_i \subseteq \Upsilon, \ i = 1, 2, \ldots\}$, with Υ_1 initialized to $\Upsilon_1 = \Upsilon$. We discuss later in this section how these subdomains can be obtained through successive *domain partitions*, i.e., by splitting the range of each relay selection variable α_r^s and each power allocation variable P_s^f and Q_r^f, with $s \in \mathcal{S}, r \in \mathcal{R}$ and $f \in \mathcal{F}$. For any subdomain Υ_i, consider $\text{UP}(\Upsilon_i)$ and $\text{LR}(\Upsilon_i)$ as an upper and lower bounds on U in (8) over subdomain Υ_i. As opposed to the global upper and lower bounds over the initial domain set Υ, we refer to $\text{UP}(\Upsilon_i)$ and $\text{LR}(\Upsilon_i)$ as local upper bound and local lower bound, respectively. Then, the global upper and lower bounds can be updated as

$$\text{UP}_{\text{glb}} = \max_i \{\text{UP}(\Upsilon_i)\} \tag{32}$$

$$\text{LR}_{\text{glb}} = \max_i \{\text{LR}(\Upsilon_i)\}. \tag{33}$$

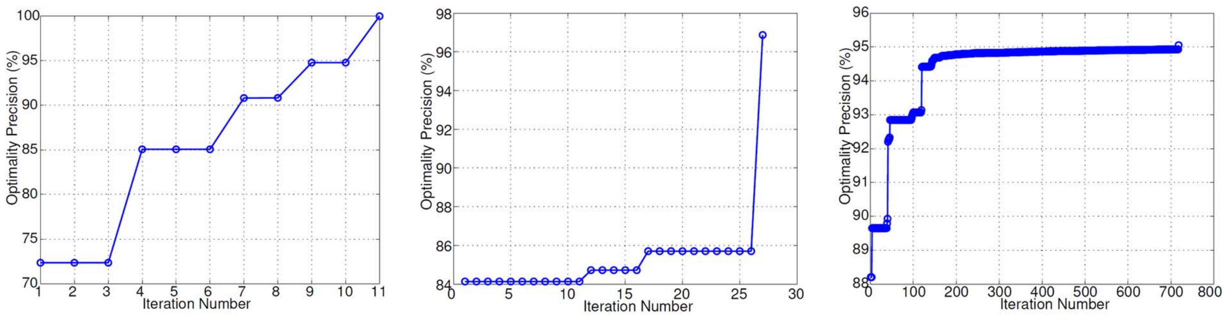
If $\text{LR}_{\text{glb}} \geq \epsilon \text{UP}_{\text{glb}}$, the algorithm terminates and sets the optimal objective U in (8) to $U = \text{LR}_{\text{glb}}$; otherwise, the algorithm chooses a subdomain from $\widetilde{\Upsilon}$ and further partitions it into two new subdomains, calculates $\text{UP}(\cdot)$ and $\text{LR}(\cdot)$, and updates the UP_{glb} and LR_{glb} as in (32) and (33). In the proposed algorithm, we select the subdomain Υ_{i^*} with the highest local upper bound from $\widetilde{\Upsilon}$, i.e., $i^* = \arg\max_i \text{UP}(\Upsilon_i)$.

Based on the update criterion of UP_{glb} and LR_{glb} in (32) and (33), the global upper and lower bounds (i.e., UP_{glb} and LR_{glb}) converge as the domain partition progresses. This can be guaranteed by the following two properties of the proposed algorithm: 1) As the domain partition (i.e., splitting a subdomain by splitting the range of its variables) progresses, the measure of each subdomain in $\widetilde{\Upsilon}$ monotonically decreases to zero and consequently the corresponding transmission strategy becomes fixed. For example, more relay selection variables become fixed to 0 or 1, and the allowed transmission power for each node becomes located within a small range; 2) As will be clearer in Section V-B, for each subdomain the corresponding nonconvex optimization problem can be relaxed to a standard convex one; with that the computed local-upper-bound objective value monotonically decreases as the subdomain becomes smaller; and finally, as variables in each subdomain become fixed, the gap between the local upper and lower bounds also shrinks to zero. From (31), UP_{glb} and LR_{glb} converge to the globally maximal objective function U^*.

The branch-and-bound framework requires that, for subdomain $\Upsilon_i \in \widetilde{\Upsilon}$, it should be computationally easy to obtain the corresponding local bounds $\text{UP}(\Upsilon_i)$ and $\text{LR}(\Upsilon_i)$. To determine $\text{UP}(\cdot)$, we rely on relaxation, i.e., we relax the original nonconvex optimization problem into a convex one that is easy to solve using standard convex programming techniques. For $\text{LR}(\cdot)$, we locally search for a *feasible solution* starting from the relaxed solution and set the corresponding sum-throughput as the local lower bound. Next, we describe how to convexify the problem formulated in (8)–(15) through relaxation.

B. Upper Bound Through Convex Relaxation

Recall that the problem formulated in Section III is a MINCoP. The objective of convex relaxation is to relax the MINCoP and its subproblems to convex problems, which can be solved easily and optimally. In the proposed algorithm, we



(a) (b) (c)

Fig. 1. Convergence of the centralized solution algorithm with network topology id (a) 1, (b) 3, and (c) 5.

consider two cases of problems: 1) if the relay selection strategy is not fixed, we solve a problem of relay selection by assuming maximum transmission power for each source and relay node and minimum interference at each destination node; otherwise 2) we solve a problem of power allocation with given fixed relay selection and minimum interference at each destination node. Next, we describe the convex relaxation in the first case as an example while the problem can be convexified similarly in the second case.

In the original MINCoP formulated in (8)–(15), all relay selection variables $\alpha_r^s,\ r \in \mathcal{R},\ s \in \mathcal{S}$ are not fixed and each α_r^s takes a value of either 0 or 1. In this case, we first relax the constraint of single relay selection by allowing each session to select multiple relay nodes and one relay node to be selected by multiple communication sessions. Then, the constraint in (9) can be rewritten as

$$0 \le \alpha_r^s \le 1 \qquad \forall s \in \mathcal{S},\ \forall r \in \mathcal{R}. \tag{34}$$

Next, we calculate an upper bound on C_{dir}^s in (1), while an upper bound on $C_{\mathrm{cop}}^{s,r}$ can be calculated similarly. To this end, denote the minimum and maximum transmission power on channel f as $P_{\min}^{s,f}$ and $P_{\max}^{s,f}$ for source node s, and $Q_{\min}^{r,f}$ and $Q_{\max}^{r,f}$ for relay node r. By substituting $P_{\min}^{s,f}$ and $Q_{\min}^{r,f}$ into (6) for $P_w^f,\ Q_r^f$, a lower bound on I_s^f can be obtained by solving a linear minimization problem over all relay selection variables, with objective function I_s^f in (6) and the feasible set of α_r^s defined in (10), (11) and (34). Denote the resulting lower bound of I_s^f as $I_{\min}^{s,f}$. Then, an upper bound on C_{dir}^s can be calculated by solving a maximization problem formulated in (2), (12) and (14) with (2) being the objective function. Note that the objective function (2), with I_s^f fixed to its lower bound $I_{\min}^{s,f}$, is concave with respect to the power allocation variables P_s^f. Hence, the optimal objective function, i.e., the objective function that provides a local upper bound to C_{dir}^s can be easily calculated. We refer to the corresponding optimal solution of $\boldsymbol{\alpha}$, denoted as $\boldsymbol{\alpha}^*$, as the relaxed solution.

VI. Performance Evaluation

We consider cooperative networks with communication area of size 1500×1500 m^2 and 750×750 m^2. Source, destination, and relay nodes are randomly placed in the area. The average channel gain between two nodes, say $m,\ n$, is determined by the distance between them, i.e.,

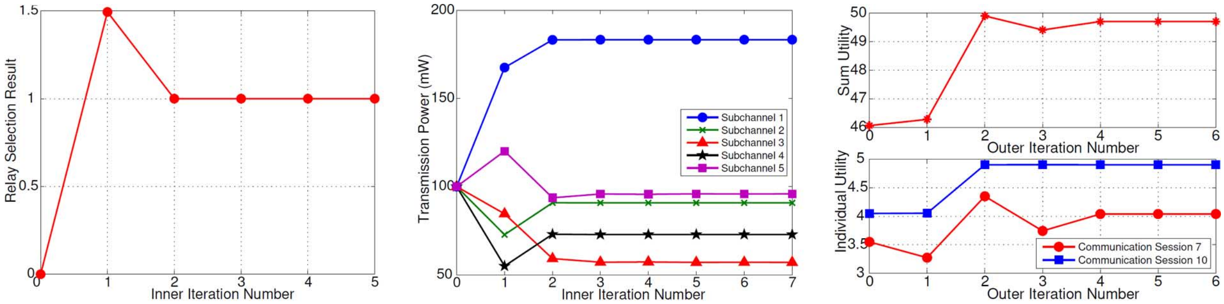
$$G_{m,n} = |d(m,n)|^{-4} \tag{35}$$

TABLE I
Network Parameters

Network Size (m^2)	id	Num. Sessions	Num. Relays	Num. Channels
1500×1500	1	2	10	4
	2	3	5	5
	3	5	5	5
	4	10	5	5
	5	10	5	2
750×750	6	$2, 4, \cdots, 10$	5	2
	7	3	$2, 4, \cdots, 10$	2

where 4 represents the path loss factor, and $d(m,n)$ represents the distance between the two nodes. The maximum transmission power for each source and relay node is set to 0.5 W. The average noise power is set to 10^{-10} W. The bandwidth of each channel is set to 64 kHz. To approximate the cooperative link capacity to a continuously differentiable function, the approximation parameters P in (19) is set to $P = 5$. Different network topology parameters are employed in our experiments to model sufficient, moderate, and scarce radio resources (relay nodes and spectrum) compared to the number of communication sessions. These parameters are summarized in Table I. In the centralized solution algorithm, optimality precision is set to $\epsilon = 95\%$. To show convergence of the proposed distributed algorithms, results are obtained using one instance of each network topology and channel realization. For performance comparison among different algorithms, results are obtained by averaging over 100 independent simulations.

Convergence of Algorithms: Since the centralized algorithm is proposed to provide a benchmark performance for distributed algorithms, convergence performance of the centralized algorithm is illustrated first. In Fig. 1, the optimality precision is defined as the ratio of global lower bound on the objective function in (7) to the global upper bound. Network topologies 1, 3, and 5 are considered. In all three cases, the proposed centralized algorithm converges to the ϵ-optimal solution, and the convergence speed of the proposed algorithm varies in different network topologies. As shown in Fig. 1(a) and (b), convergence is very fast in case of small number of communication sessions, e.g., in network topologies 1 and 3. When the number of communication sessions is high and hence the level of interference is high, e.g., in network topology 5, the proposed algorithm requires more iterations to converge as shown in Fig. 1(c). However, the algorithm still converges very quickly to a 93% level of optimality. Therefore, it is possible to trade precision for computation time.



(a) (b) (c)

Fig. 2. Convergence of the decomposition-based solution algorithm with network topology id 4. (a) Relay selection; values greater than 1 means that the relay is selected by multiple communication sessions. (b) Power allocation. (c) Sum utility (top) and individual utility (bottom).

The convergence performance of the decomposition-based solution algorithm is illustrated in Fig. 2.[4] Network topology 4 is used in the simulation. In such a network topology, the number of relay nodes and also channels is less than, but comparable to, the number of concurrent communication sessions. Convergence of the distributed relay selection (DRS) algorithm at a relay node is shown in Fig. 2(a) with uniform power allocation over different channels. All relay selection variables are initialized to zero, i.e., no transmitter selects the relay node for cooperative relaying. After one iteration, at least two transmitters select the relay node. Notice that a transmitter might choose to use a relay node for only part time of a transmission, but use other relay nodes or direct link for the rest part. Consequently, the relay selection constraint (11) becomes violated for this relay node and he increases his price. As a result, the relay selection converges with all constraints in (11) satisfied. We can see that the proposed penalization-based algorithm converges very fast. Distributed spectrum management (DSM) by power allocation is shown in Fig. 2(b). In power allocation, results of the relay selection obtained in Fig. 2(a) are employed. Here, power allocation for a source node over multiple channels is shown as an example. We can see that the proposed best-response algorithm converges within three iterations. Results of joint DRS and DSM are given in Fig. 2(c). The top figure shows the sum utility of all communication sessions, while the bottom figure shows individual utilities for two communication sessions. Individual utilities become stable after four iterations of DRS or DSM. To summarize, the proposed algorithms of DRS, DSM, and joint DRS and DSM have a good convergence performance.

Optimality Analysis: Performance of the distributed solution algorithms is evaluated by comparing them to the ϵ-optimal solution achieved by the centralized algorithm. In Fig. 3, the sum utility achievable by the centralized and the two distributed solution algorithms are plotted for 100 simulation instances, considering network topology 2 with communication area of 1500×1500 m^2. We can see that the distributed algorithms achieve sum utility very close to that of the centralized in most tested cases; the decomposition-based distributed solution algorithm achieves in general the same or slightly higher sum utility than the transformation-based algorithm.

The average performance of the three algorithms is reported in Fig. 4 considering network topologies 1–5, in terms of sum utility in Fig. 4(a) and in terms of the corresponding sum capacity in Fig. 4(b). We can see from Fig. 4(a) that the decomposition-based distributed solution algorithm achieves around 95.05%–96.88% of that of the centralized in terms of sum utility, with an average optimality around 96%; around 92.34%–94.73% can be achieved by the transformation-based distributed solution algorithm with an average of 93.82%. The corresponding average performance in terms of sum capacity is shown in Fig. 4(b); the decomposition-based algorithm achieves 89.48%–96.32% of that of the centralized, while 85.48%–88.78% by the transformation-based solution algorithm. Comparing the two distributed solution algorithms, we found that integrated resource allocation (i.e., without decomposing the original optimization problem) may not necessarily lead to higher social utility, i.e., sum utility or sum capacity in this work.

[4]As mentioned earlier, the transformation-based algorithm is not guaranteed to converge, and we omit the corresponding discussion because of space limits.

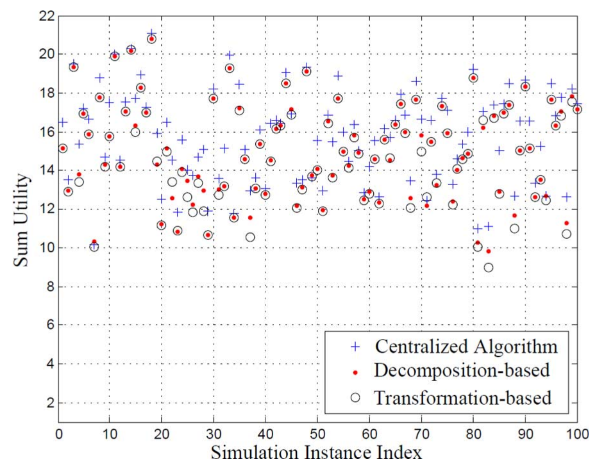


Fig. 3. Sum utility achievable by the centralized and the two distributed solution algorithms in 100 simulation instances.

In Fig. 5, the average sum utility is plotted against the number of sessions in Fig. 5(a) and the number of relay nodes in Fig. 5(b), considering network topologies 6 and 7 with communication area of 750×750 m^2. Since smaller communication area implies higher user density, and hence higher interference among them, it may potentially degrade the performance of the distributed solution algorithms. In this case, the decomposition-based solution algorithm achieves 92.39%–95.78% sum utility of the centralized with different number of sessions. By varying the number of relays, the achievable sum utility

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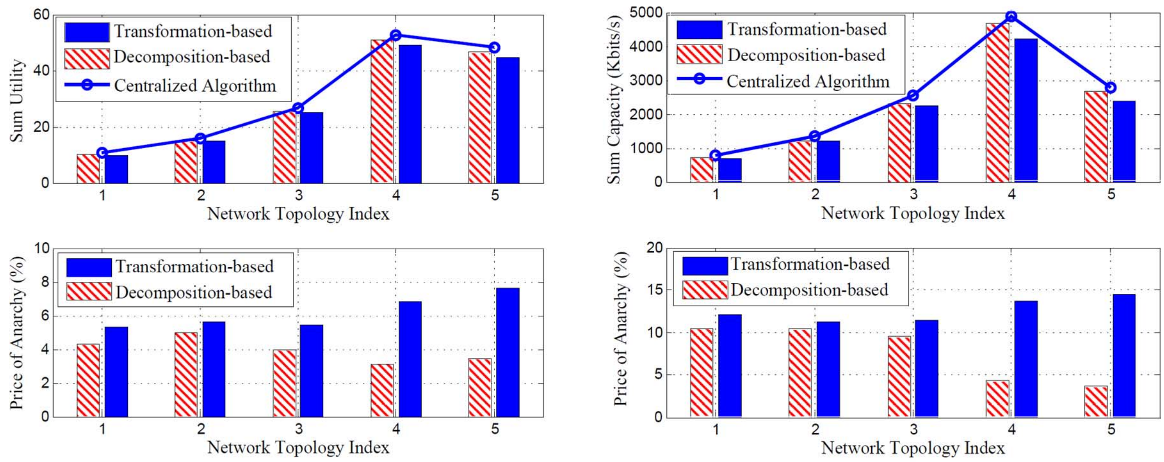
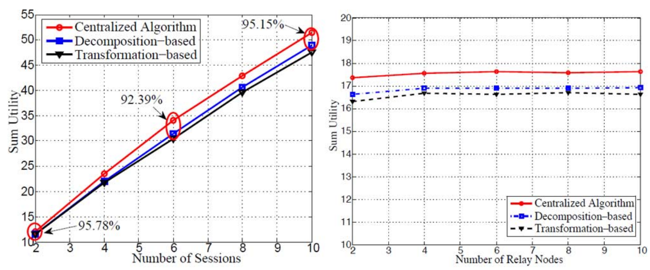


(a) (b)

Fig. 4. (a) Average sum utility achievable by the centralized and the two distributed solution algorithms and the corresponding price of anarchy. (b) Corresponding average sum capacity and price of anarchy.



(a) (b)

Fig. 5. Average sum utility achievable by the centralized and the two distributed solution algorithms with (a) different number of sessions and (b) different number of relay nodes.

changes only slightly, with good performance still achievable by the two distributed solution algorithms. In the case of even higher interference, it would be beneficial to introduce signaling exchange among the sessions so that they can decide their power allocation strategies cooperatively in favor of higher social utility [50].

VII. Conclusion

In this paper, we have studied distributed spectrum management and relay selection in cognitive and cooperative wireless networks. We first formulated the problem of joint spectrum management and relay selection, and then decomposed it into two individual problems: 1) spectrum management by power allocation with given fixed relay selection, and 2) relay selection with given fixed spectrum profile. A distributed solution algorithm is proposed for each subproblem and analyzed based on the variational inequality (VI) theory. We prove that the proposed algorithms converge to a VI solution, which is also an NE solution. Performance of the distributed algorithm is evaluated by comparing to the centralized solution. Simulation results indicate that the distributed algorithm has performance that is very close to the optimal solution. Convergence of the distributed algorithm is also verified using simulation results. The distributed algorithm can be used to schedule an independent set of transmissions, each of which is scheduled by a separate algorithm.

Appendix

A. Proof of Sufficient Condition

In our VI problem, the domain set $\mathcal{X}$ is closed and convex. From Lemma 1, we have that $\mathrm{VI}(\mathcal{X}, \boldsymbol{F})$ has at least one solution. From Lemma 2, we have that the utility function $U_s(\boldsymbol{x}_s, \boldsymbol{x}_{-s})$ is strongly concave for each player given fixed transmission strategies. Then, the strong concavity of $U_s(\boldsymbol{x}_s, \boldsymbol{x}_{-s})$ implies that $F_s = \nabla_{\boldsymbol{x}_s} U_s$ is strongly monotonic [46]. In the following, we present a sufficient condition for the most complicated case, i.e., when each session uses a cooperative relay, that can be derived based on the framework in [52]. The resulting sufficient condition can be easily extended to other cases.

If relay node r, with $r \in \mathcal{R}$, is selected by session s, with $s \in \mathcal{S}$, then the gradient vector of session s with respect to $\boldsymbol{x}_s$ can be written as $J_{\boldsymbol{x}_s}(U_s) = \left(\left(\frac{\partial U_s}{\partial P_s^f}\right)_{f=1}^F, \left(\frac{\partial U_s}{\partial Q_r^f}\right)_{f=1}^F \right)$, where U_s is utility function of session s defined in (7), $\boldsymbol{x}_s = ((P_s^f)_{f=1}^F, (Q_r^f)_{f=1}^F)$ is the power allocation vector of session s, and F represents the number of subchannels in set $\mathcal{F}$. Similarly, we represent the gradient vector of session g, with $g \in \mathcal{S}$, as $J_{\boldsymbol{x}_g}(U_g) = \left(\left(\frac{\partial U_g}{\partial P_g^f}\right)_{f=1}^F, \left(\frac{\partial U_g}{\partial Q_t^f}\right)_{f=1}^F \right)$, where t is the relay node selected by session g. Further, denote the Jacobi matrix of $J_{\boldsymbol{x}_s}(U_s)$ and $J_{\boldsymbol{x}_g}(U_g)$ with respect to x_s as $J_{\boldsymbol{x}_s\boldsymbol{x}_s}(U_s)$ and $J_{\boldsymbol{x}_g\boldsymbol{x}_s}(U_g)$, respectively. Here, $J_{\boldsymbol{x}_s\boldsymbol{x}_s}(U_s)$ is namely the Hessian matrix session s. Then, we can define a matrix $[\gamma]_{ij}$ as follows:

$$[\gamma]_{sg} \triangleq \begin{cases} \alpha_s^{\min}, & \text{if } s = g \\ -\beta_{sg}^{\max}, & \text{otherwise} \end{cases} \tag{36}$$

where $\alpha_s^{\min} \triangleq \inf_{x \in \mathcal{X}} \lambda_{\text{least}}(J_{\boldsymbol{x}_s\boldsymbol{x}_s}(U_s))$ and $\beta_{sg}^{\max} \triangleq \sup_{x \in \mathcal{X}} \|J_{\boldsymbol{x}_g\boldsymbol{x}_s}(U_g)\|$, with $\lambda_{\text{least}}(\boldsymbol{A})$ representing the eigenvalue of $\boldsymbol{A}$ with the smallest absolute value. Then, based on the properties of the P-matrix [53], to guarantee the convergence of the proposed distributed algorithm, we only need to show that the matrix $[\gamma]_{sg}$ defined in (36) is a P-matrix, which follows the conditions in (21)–(23) [52].

$$\left.\begin{aligned} \textstyle\sum_{r\in\mathcal{R}_G}\lambda_r\nabla G_r(\boldsymbol{\alpha})+\sum_{r\in\widehat{\mathcal{R}},s\in\widehat{\mathcal{S}}}\widehat{\mu}_r^s\nabla\widehat{H}_r^s(\boldsymbol{\alpha})+\sum_{r\in\widetilde{\mathcal{R}},s\in\widetilde{\mathcal{S}}}\widetilde{\mu}_r^s\nabla\widetilde{H}_r^s(\boldsymbol{\alpha})+\sum_{s\in\overline{\mathcal{S}}}\overline{\mu}^s\nabla\overline{H}^s(\boldsymbol{\alpha}) \\ \lambda_r\geq 0,\ \forall r\in\mathcal{R}_G \\ \widehat{\mu}_r^s\geq 0,\ \forall r\in\widehat{\mathcal{R}},\ s\in\widehat{\mathcal{S}} \\ \widetilde{\mu}_r^s\geq 0,\ \forall r\in\widetilde{\mathcal{R}},\ s\in\widetilde{\mathcal{S}} \\ \overline{\mu}^s\geq 0,\ \forall s\in\overline{\mathcal{S}} \end{aligned}\right\} \Rightarrow \lambda_r=\widehat{\mu}_r^s=\widetilde{\mu}_r^s=\overline{\mu}^s=0,\ \forall(r,s)\in\Phi_G\cup\widehat{\Phi}\cup\widetilde{\Phi}\cup\overline{\Phi} \tag{45}$$

B. Proof of Convergence of DRS Algorithm

To prove *Lemma 5*, it is equivalent to prove the following two statements: 1) Algorithm 2 converges as iteration index k tends to ∞; and 2) every convergence point is a VI solution of $\mathrm{QVI}(\Phi,\boldsymbol{F})$ formulated in Section IV-C.2.

First, we represent domain set Φ defined in (9)–(11) using a new set of functions as follows:

$$\widehat{H}_r^s(\boldsymbol{\alpha})\leq 0 \qquad \forall r\in\mathcal{R},\ s\in\mathcal{S} \tag{37}$$
$$\widetilde{H}_r^s(\boldsymbol{\alpha})\leq 0 \qquad \forall r\in\mathcal{R},\ s\in\mathcal{S} \tag{38}$$
$$\overline{H}^s(\boldsymbol{\alpha})\leq 0 \qquad \forall s\in\mathcal{S} \tag{39}$$
$$G_r(\boldsymbol{\alpha})\leq 0 \qquad \forall r\in\mathcal{R} \tag{40}$$

where $\boldsymbol{\alpha}=(\alpha_r^s)_{r\in\mathcal{R}}^{s\in\mathcal{S}}$ is the relay selection profile of all sessions defined in Section III; functions $\widehat{H}_{r,s}$, $\widetilde{H}_{r,s}$, $\overline{H}_s$, and G_r are defined as

$$\widehat{H}_r^s(\boldsymbol{\alpha})=-\alpha_r^s \qquad \forall r\in\mathcal{R},\ s\in\mathcal{S} \tag{41}$$
$$\widetilde{H}_r^s(\boldsymbol{\alpha})=\alpha_r^s-1 \qquad \forall r\in\mathcal{R},\ s\in\mathcal{S} \tag{42}$$
$$\overline{H}^s(\boldsymbol{\alpha})=\sum_{r\in\mathcal{R}}\alpha_r^s-1 \qquad \forall s\in\mathcal{S} \tag{43}$$
$$G_r(\boldsymbol{\alpha})=\sum_{s\in\mathcal{S}}\alpha_r^s-1 \qquad \forall r\in\mathcal{R} \tag{44}$$

respectively. Here, constraints (37) and (38) together correspond to constraint (9), and (39) and (40) correspond to (10) and (11), respectively. Constraints (37)–(39) impose a constraint on the relay selection strategy for each session $s\in\mathcal{S}$, while (40) imposes a coupled constraint for all the sessions in $\mathcal{R}$.

Next, we show that the implication in (45) at the top of the page holds for any $\boldsymbol{\alpha}\in\Phi$, where $\Phi_G\equiv\{(r,s):G_r(\boldsymbol{\alpha})\geq 0\}$, $\widehat{\Phi}\equiv\{(r,s):\widehat{H}_r^s(\boldsymbol{\alpha})\geq 0\}$, $\widetilde{\Phi}\equiv\{(r,s):\widetilde{H}_r^s(\boldsymbol{\alpha})\geq 0\}$, and $\overline{\Phi}\equiv\{(r,s):\overline{H}^s(\boldsymbol{\alpha})\geq 0\}$. For this purpose, we investigate the gradient vector of $G_r(\boldsymbol{\alpha})$, $\widetilde{H}_r^s(\boldsymbol{\alpha})$, $\overline{H}^s(\boldsymbol{\alpha})$, and $\widehat{H}_r^s(\boldsymbol{\alpha})$ with respect to the relay selection profile $\boldsymbol{\alpha}$. We have the following observations, where $|\mathcal{R}|$ and $|\mathcal{S}|$ represnt the cardinality of set $\mathcal{R}$ and $\mathcal{S}$, respectively.

- For each $G_r(\boldsymbol{\alpha})$, $\widetilde{H}_r^s(\boldsymbol{\alpha})$, or $\overline{H}^s(\boldsymbol{\alpha})$, the gradient is a $|R|\cdot|S|$ dimensional vector, with each element being either 1 or 0, and at least one element being 1. For example, the gradient vector of $\widetilde{H}_r^s(\boldsymbol{\alpha})$ can be given as $(0,\ldots,0,1,0,\ldots,0)^{\mathrm{T}}$, where the single 1-element corresponds to that the r^{th} relay node is selected by the s^{th} session.
- For each $\widehat{H}_r^s(\boldsymbol{\alpha})$, the gradient vector can be given as $(0,\ldots,0,-1,0,\ldots,0)^{\mathrm{T}}$, where the single -1-element corresponds to that the r^{th} relay node is not selected by the s^{th} session.

Since it is impossible for all tuples (r,s) to be included in $\widehat{\Phi}$ (no relay node selected by any sessions) while at the same time some tuples (r,s) are also included in $\widetilde{\Phi}$ or Φ_G (some relay nodes selected by some sessions), the above two observations imply that the gradient vectors in (45) are linearly independent, which proves the implication.

Then, it follows immediately [54, Theorem 3] that *the limit of any convergent subsequence* $\{\boldsymbol{\alpha}_k\}$ *generated by Algorithm 2, denoted by* $\boldsymbol{\alpha}_\infty$*, is a VI solution of* $\mathrm{QVI}(\Phi,\boldsymbol{F})$; moreover, the sequence $\{u_k^r\}$ generated using (27) is bounded, implying that the penalization item in (24) tends to 0 as $\rho_k\to\infty$. Hence, the second statement at the opening of the proof holds.

Finally, the bounded $\{u_k^r\}$ implies that $\boldsymbol{\alpha}_\infty$ is also the limit of the entire sequence since: *1) for sufficiently large iteration index k, the modified utility in* (24) *reduces to the original utility as the penalization vanished; 2) any solution of* $\mathrm{QVI}(\Phi,\boldsymbol{F})$ *is a Nash equilibrium of the relay selection game* $\mathrm{GNE}(\Phi,\boldsymbol{U})$ *formulated in* Section IV-C*; and 3) at an NE point no session has incentives to unilaterally deviate from its current relay selection strategy, and otherwise its (original) utility will be decreased*. Therefore, the first statement also holds.

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Zhangyu Guan (M'11) received the Ph.D. degree in communication and information systems from Shandong University, Jinan, China, in 2010.

He is currently a Postdoctoral Research Associate with the Department of Electrical and Computer Engineering, Northeastern University, Boston, MA, USA. He was a visiting Ph.D. student with the Department of Electrical Engineering, The State University of New York (SUNY) at Buffalo, Buffalo, NY, USA, from 2009 to 2010. He was a Lecturer with Shandong University from 2011 to 2014. He was a Postdoctoral Research Associate with the Department of Electrical Engineering, SUNY Buffalo, from 2012 to 2015. His current research interests are in cognitive radio and software-defined networking, wireless multimedia sensor networks, and underwater networks.

Dr. Guan has served as a TPC member for IEEE INFOCOM 2016, IEEE GLOBECOM 2015, IEEE ICNC 2012–2015, and IEEE VTC 2011-Fall and 2015-Fall, among others, and served as a reviewer for the IEEE TRANSACTIONS ON COMMUNICATIONS, the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, and the IEEE COMMUNICATIONS SURVEYS AND TUTORIALS, among others.

Tommaso Melodia (M'07) received the Ph.D. degree in electrical and computer engineering from the Georgia Institute of Technology, Atlanta, GA, USA, in 2007.

He is an Associate Professor with the Department of Electrical and Computer Engineering, Northeastern University, Boston, MA, USA. His research has been supported by the National Science Foundation, Air Force Research Laboratory, and the Office of Naval Research, among others. His current research interests are in modeling, optimization, and experimental evaluation of networked communication systems, with applications to ultrasonic intrabody networks, cognitive and cooperative networks, multimedia sensor networks, and underwater networks.

Prof. Melodia was the Technical Program Committee Vice Chair for IEEE GLOBECOM 2013 and the Technical Program Committee Vice Chair for Information Systems for IEEE INFOCOM 2013. He serves on the editorial boards of the IEEE TRANSACTIONS ON MOBILE COMPUTING, the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, the IEEE TRANSACTIONS ON MULTIMEDIA, and *Computer Networks*. He was a recipient of the National Science Foundation CAREER Award and coauthored a paper that was recognized as the ISI Fast Breaking Paper in the field of Computer Science for February 2009 and of an ACM WUWNet 2013 Best Paper Award.

Dongfeng Yuan (SM'01) received the M.S. degree from Shandong University, Jinan, China, in 1988, and the Ph.D. degree from Tsinghua University, Beijing, China, in 2000, both in electrical engineering.

Currently, he is a Full Professor with the School of Information Science and Engineering, Shandong University, China. From 1993 to 1994, he was with the Electrical and Computer Department, University of Calgary, Calgary, AB, Canada. He was with the Department of Electrical Engineering, University of Erlangen, Erlangen, Germany, from 1998 to 1999; the Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI, USA, from 2001 to 2002; the Department of Electrical Engineering, Munich University of Technology, Munich, Germany, in 2005; and the Department of Electrical Engineering, Heriot-Watt University, Edinburgh, U.K., in 2006. He has published over 300 technical papers in the international journals and the proceedings of some important international conferences. His research interests are in the areas of wireless communication and networks, including coding theory, cross-layer design, and techniques toward 5G wireless communications.

Dimitris A. Pados (M'95–SM'15) was born in Athens, Greece, in 1966. He received the Diploma degree in computer science and engineering (five-year program) from the University of Patras, Patras, Greece, in 1989, and the Ph.D. degree in electrical engineering from the University of Virginia, Charlottesville, VA, USA, in 1994.

From 1994 to 1997, he was an Assistant Professor with the Department of Electrical and Computer Engineering and the Center for Telecommunications Studies, University of Louisiana, Lafayette, LA, USA. Since 1997, he has been with the Department of Electrical Engineering, The State University of New York at Buffalo, Buffalo, NY, USA, holding in sequence the titles of Assistant Professor, Associate Professor, Professor, and currently Clifford C. Furnas Eminent Professor of Electrical Engineering. He served the department as Associate Chair from 2009 to 2010. He was elected three times University Faculty Senator (terms 2004–2006, 2008–2010, 2010–2012) and served on the Faculty Senate Executive Committee in 2009 to 2010. His research interests are in the general areas of communication theory and systems and adaptive signal processing with applications to interference channels and signal waveform design, secure wireless communications, cognitive radios, and networks.

Dr. Pados is a member of the IEEE Communications, Signal Processing, Information Theory, and Computational Intelligence societies. He served as an Associate Editor for the IEEE SIGNAL PROCESSING LETTERS from 2001 to 2004 and the IEEE TRANSACTIONS ON NEURAL NETWORKS from 2001 to 2005. He received a 2001 IEEE International Conference on Telecommunications Best Paper Award, the 2003 IEEE TRANSACTIONS ON NEURAL NETWORKS Outstanding Paper Award, the 2010 IEEE International Communications Conference Best Paper Award in Signal Processing for Communications, the 2013 International Symposium on Wireless Communication Systems Best Paper Award in Physical Layer Communications and Signal Processing, and a Best of IEEE GLOBECOM 2014—Top 50 Papers distinction for articles he coauthored with students and colleagues. He is a recipient of the 2009 SUNY-system-wide Chancellor's Award for Excellence in Teaching and the 2011 University at Buffalo Exceptional Scholar—Sustained Achievement Award.